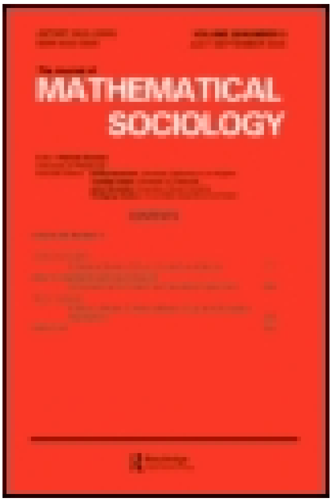

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Online and Offline Social Participation and Social Poverty Traps: Can Social Networks Save Human Relations?

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In this study, we develop an evolutionary game model to analyze how human relations evolve in a context characterized by declining face-to-face interactions and growing online social participation. Our results suggest that online networks may constitute a coping response allowing individuals to "defend" their social life from increasing busyness and a reduction in the time available for leisure. Internet-mediated interaction can play a positive role in preventing the disruption of ties and the weakening of community life documented by empirical studies. In this scenario, the digital divide is likely to become an increasingly relevant factor of social exclusion, which may exacerbate inequalities in well-being and capabilities.

Keywords: digital divide, online networks, social capital, social participation, well-being

1. INTRODUCTION

Economic development and technological progress continuously and profoundly transform our way of living. Recently, the advent of the Internet and online networks has made economic and cultural changes even more radical and rapid, causing a major shift toward new types of social interaction. Studies in the social capital literature have documented two stylized facts: first, indicators of social participation declined in the years that preceded the social networking revolution (Bartolini, Bilancini, & Pugno, 2013; Costa & Kahn, 2003; Putnam, 2002; Sarracino, 2010), and second, in the last 5 years the success of social networking sites (SNSs) has resulted in a steep rise in online social participation (Duggan & Brenner,

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2013; Smith, 2013). Despite the immensity of these transformations, we still lack a systematic theoretical analysis of the role of online interactions in the evolution of human relations. It is also not clear whether Internet usage and SNSs may accelerate the decline in social participation, or if they offer a way to support social relationships against the threats posed by the disruption of ties and the weakening of community life documented by empirical studies.

In this article, we draw on the empirical literature on social capital and computer-mediated communication to develop a theoretical framework for an analysis of how human relations may evolve in a context characterized by declining face-to-face interactions and growing online social participation. More specifically, we aim to deepen our understanding of (a) the circumstances that may exacerbate the decline in social participation, (b) the extent to which Internet-mediated interaction can help in preventing "social poverty traps" (Antoci, Sacco, & Vanin, 2007), and (c) how the decline in social participation affects the relative performance of the different types of participation.

In our framework, agents can choose to participate socially or to withdraw from social relations, or at least reduce them to the minimum, in order to devote all their available time to work. Following Antoci, Sabatini, and Sodini (2012a, 2013), we assume that individuals who participate socially develop interactions in two ways:

1. Using a "social networking strategy" (hereafter *SPN*, Social Participation through online Networks), within which social participation takes place *both* by means of online networking and face-to-face interactions. This strategy entails different degrees of Internet-mediated interaction according to individual preferences and the characteristics of their reference groups: in general, we think of *SPN* agents as individuals who develop social ties online at their convenience—for example, by staying in touch with friends and acquaintances, or interacting with unknown others, through SNSs—and meet their contacts in person whenever they want or have time. On the other hand, at the extreme end of this strategy, *SPN* agents can develop an entire social life online, as in the case of Japanese *hikikomori*.[1] These agents will never physically meet their Facebook friends.

[1]The Japanese term *hikikomori* refers to young people who have withdrawn from social life and have had no relationships outside of the family for a period of more than 6 months. After its rise in Japan, this phenomenon has been increasingly observed in other developed and developing countries (Kato et al., 2012). *Hikikomoris* do not work or participate in any form of education and frequently remain in their homes for protracted periods of time—sometimes for several years. From a psychological perspective that has long dominated public thinking in Japan, *hikikomoris* suffer from a cognitive malfunction. The sociological perspective, however, advances a more interesting interpretation of the phenomenon, as a form of anomie related to the nature of family relations and a breakdown in social and labor opportunity structures. In other words, the phenomenon can also be viewed as a reaction to the "relational poverty" of the social environment and to the lack of proper opportunities of social and labor participation (Furlong, 2008). According to Kaneko (2006), *hikikomoris* may be understood to be reacting to time pressures and role performances in modern societies. Before the advent of online networks, *hikikomoris* had no relevant social interaction. With the advent of the Internet, psychologists have observed resurgence in the social relationships of *hikikomoris*, who generally tend to have numerous online interactions with others (Kato et al., 2012). The case of *hikikomoris* will be useful for explaining our assumptions in Section 3.

3. As an alternative, socially participating individuals can choose not to use Internet-mediated communication and to develop an entire social life through face-to-face encounters. We call this strategy *SPF* (Social Participation through Face-to-face encounters). These strategies can be seen as two alternative *technologies* of social participation, represented by vectors (s_1, f_1) and (s_2, f_2), where s_1 and s_2 are the time devoted to online social participation by individuals respectively playing the *SPN* and the *SPF* strategy and f_1 and f_2 represent the time devoted to face-to-face social participation by *SPN* and *SPF* players respectively, $s_1 > s_2 = 0$, $f_2 > f_1 \geq 0$ and $s_1 + f_1 = s_2 + f_2$.

Unlike Antoci et al. (2012a, 2013), we specifically consider the choice of social isolation by agents who prefer to devote all their time to work and to forms of *private* consumption[2] that do not entail any significant relationships. We call this third strategy *NSP* (No Social Participation). It is worth noting that, in our framework, this withdrawal from social relations does not imply retirement from work (as for *hikikomori*). In an *NSP* agent's way of life, social relations are kept to a necessary minimum, and on-the-job interactions do not entail the formation of friendships. *NSP* individuals tend to replace relational goods (e.g., playing a match on a football field with 21 friends) with material goods (e.g., playing a virtual match at home on a PlayStation) in their consumption choices.

The analysis shows that, depending on the configuration of payoffs, the state where all individuals play *NSP* can be locally attractive; that is, it constitutes a social poverty trap where relational goods are produced and consumed in an extremely small amount and nobody participates in social activities. However, if the social networking strategy is rewarding enough in respect to the withdrawal from participation, then the state where all agents choose *NSP* becomes a saddle point. In this case, the use of the Internet for preserving and developing social ties avoids the social poverty trap, and the community reaches a steady state where all agents adopt the *SPN* strategy (which, under a certain configuration of payoffs, may also be globally attractive). This state, however, may be Pareto-dominated by the alternative state where all agents play *SPF*, and can therefore be viewed as a second-best scenario.

Our results suggest that the "*SPN* way of life" may be interpreted as a coping response with which agents "react" to increasing busyness, a reduction in the time available for leisure, and the cultural and relational impoverishment of the social environment. From this point of view, this article proposes a framework to better understand results from the empirical studies finding a complementarity between online and offline interactions, which suggested that forms of Internet-mediated communication can play a crucial role in preventing the disruption of ties and the weakening of community life (Ellison, Steinfield, & Lampe, 2007; Sabatini & Sarracino, 2014a). This argument implies that individuals who do not use the Internet (e.g., because they do not have broadband access) may increasingly face problems of social integration. In this scenario, the digital divide is likely to become

[2]The literature on relational goods distinguishes private consumption that can be enjoyed alone without the inclusion of any significant social interaction and relational consumption, which can be enjoyed only if shared with others Gui & Sugden, 2005; Uhlaner, 1989).

a factor in social exclusion, which may exacerbate inequalities in well-being as well as capabilities. The literature has shown that social interactions and the accumulation of social capital positively influence subjective well-being (Becchetti, Pelloni, & Rossetti, 2008; Bruni & Stanca, 2008; Beja, 2014), health (Yamamura, 2011; Rocco, Fumagalli, & Suhreke, 2014; Fiorillo & Sabatini, 2015), education (Misra, Grimes, & Rogers, 2013), employability (Fugate, Kinicki, & Ashforth, 2004; McDonald, 2011), economic welfare and social mobility (Degli Antoni, 2009; Frey, Gallus, & Steiner, 2014), entrepreneurship (Alexy, Block, Sandner, & Yer Wal, 2012; Santarelli & Tran, 2013; Sabatini, Modena, & Tortia, 2014), and access to credit (Shoji, Aoyagi, Kasahara, Sawada, & Ueyama, 2012).

The outline of the article is as follows. Section 2 surveys the related literature. Section 3 describes our evolutionary game model. Section 4 briefly accounts for scheduling problems. Section 5 analyzes the dynamics of the model. Section 6 is devoted to a well-being analysis. The article closes with some concluding remarks and implications for future research, followed by a mathematical appendix.

2. RELATED LITERATURE

In his best-seller *Bowling Alone*, Robert Putnam (2000) draws on various sources to document that a decline in social participation measures, such as membership in formal organizations, the intensity of members' participation, informal social connectedness, and interpersonal trust, began in the United States in the 1960s and 1970s with a sharp acceleration in the 1980s and 1990s.

The "decline of community life thesis" (Paxton, 1999, p. 88) advanced by Putnam prompted a number of subsequent empirical tests. Based on General Social Surveys (GSS) data for the period 1975–1994, Paxton (1999) finds some decline in the general measure of social capital (given by a combination of trust and membership in associations), a decline in interpersonal trust, and no decline in associations. Costa and Kahn (2003) use a number of different sources to assess the development of social capital in the United States since 1952 by evaluating trends in participation and community life. The authors argue that the rise in female labor force participation and income inequality are two of the primary explanations for the decline in social capital as measured by the indicators of volunteering, membership in organizations, and entertainment with friends and relatives. Bartolini et al. (2013) use GSS data to investigate the evolution of social connections—measured through membership in Putnam and Olson groups[3] and the indicators of perceived trustworthiness,

[3]Following Knack and Keefer (1997), the literature generally distinguishes two types of formal organizations, labelled "Olsonian" and "Putnam-esque" associations. Olson groups are those associations with redistributive goals that lobby for the protection of their members' interests, possibly against the interests of other groups (Olson, 1965, 1982). Examples of this type of organization are professional and entrepreneurial associations, trade unions, and associations for the protection of consumers' rights. Putnam groups are those associations least likely to act as "distributional coalitions but which involve social interactions that can build trust and cooperative habits" (Knack & Keefer, 1997, p. 1273). Examples of this type of organization are cultural circles, sport clubs, youth associations (e.g., scouts), and religious organizations.

helpfulness and fairness, and confidence in institutions in the United States between 1975 and 2002, finding that they generally show a declining trend.

Apart from the United States, there seems to be a common pattern of declining trust, political participation, and organizational activity across industrialized democracies during the 1980s and 1990s, with the exception of China, Japan, Korea, and the Scandinavian countries (Chen & Gao, 2013; Lee, 2008; Listhaug & Grønflaten, 2007). Declining trends of one or more dimensions of social capital have been documented for England and Wales over the period 1972–1999 (Li, Savage, & Pickles, 2003), Great Britain over 1980–2000 (Sarracino, 2010), and Australia over 1960–1990 (Cox, 2002).[4]

In *Bowling Alone*, Putnam (2000) discusses three main explanations for the decline in American social capital: (a) the reduction in the time available for social interaction—related to the need to work more, to the rise in labor flexibility, and to the expansion in commuting time; (b) the rise in mobility of workers and students; and (c) technology and mass media.

In the last decade, Putnam's claims have found support in a number of studies investigating the effect exerted on various dimensions of social connectedness by the rise in working time (Bartolini & Bilancini, 2011), labor mobility (Routledge & von Amsberg, 2003), urban sprawl and commuting (Besser, Marcus, & Frumkin, 2008),[5] the psychological distress related to the need to satisfy society's expectations (Kaneko, 2006), and to the social poverty of the surrounding environment, which can prompt individuals to pursue social isolation, as in the case of Japanese *hikikomoris* (Furlong, 2008).

As for the third argument, the role of technology and media in the evolution of social interaction is widely debated in the literature. Putnam advanced the hypothesis that technological progress may be partially responsible for the erosion of American social capital at the end of the 1990s. This hypothesis was formulated just a few years before the "explosion" of the Internet and online networks. The author's explanation of the possibly negative role of technology was centered on the socially detrimental effects of television and other forms of "private" entertainment, such as video games. This argument found support in several empirical studies proving the negative influence of television on social relations (Bruni & Stanca, 2006, 2008).

[4]Despite the many studies documenting the decline in social participation, the overall evidence still seems to be nonconclusive. A number of empirical studies have found conflicting results on the trends of different indicators of social capital, and the *Bowling Alone* thesis has been variously characterized as plainly wrong, pessimistic, or traditional (Stolle & Hooghe, 2005). Van Ingen and Dekker (2011) argue that the decline in associational participation may be related to a process of "informalization" of social activities. In his cross-country analysis of social capital trends, Sarracino (2010) finds that in most Western European countries, several measures of connectedness experienced a growth over the period 1980–2000.

[5]There is different evidence on the social effects of commuting outside of the United States. In countries where cities are, on average, significantly smaller than in the United States, Putnam's thesis seems not to be supported. A Swiss study by Viry, Kaufmann, and Widmer (2009) conclude that while commuting decreases the availability of emotionally bonding social capital in the form of supportive strong ties, it could provide increased opportunities for developing bridging social capital and weak ties.

All the studies mentioned above exclusively refer to face-to-face interactions and completely disregard online participation. However, in the past few years, Internet-mediated interaction has literally revolutionized individuals' social lives. In contrast to the early age of the Internet, when being connected was predominantly an individual entertainment activity such as watching TV or reading newspapers, today, the use of the Web is strongly related to participation in SNSs, which in turn entails a variety of forms of engagement in social activities.

According to the Pew Research Center Internet & American Life Project Post-Election Survey, as of December 2012, 67% of U.S. Internet users were active on SNSs. More than four fifths of online young adults (aged 18–29) and 77% of middle-aged adults (30–49) use SNSs (Duggan & Brenner, 2013). According to a survey conducted by Princeton Survey Research Associates International in November 2010, among a sample of 2,255 adults, SNSs are used increasingly to keep up with close social ties; the average user of an SNS has more close ties and is half as likely to be socially isolated as the average American; and Facebook users have more close relationships and are much more politically engaged than the average American. Internet users get more support from their social ties than those who do not use the Internet, Facebook users get the most support and Facebook seems to play a crucial role in reviving "dormant" relationships (Brenner, 2013; Hampton, Goulet, Rainie, & Purcell, 2011). Almost half of Internet users create and share original content online. As of August 2012, 46% of adult users post original photos or videos online that they themselves have created (Brenner, 2013). Sharing is also a way to keep loved ones posted on personal experiences, which proves particularly effective, for example, for workers and students living away from home. In this case, social interactions mostly take place asynchronously. In December 2010, U.S. Internet users were found to be more likely than others to be active in some kind of voluntary group or organization: 80% of American Internet users participated in groups compared with 56% of non-Internet users. Moreover, social media users are even more likely to be active: 82% of social network users and 85% of Twitter users are group participants (Hampton et al., 2011).

These figures suggest to mitigate the fear of social isolation that the common wisdom generally associates with intense Internet usage. Several authors from different fields have begun to empirically analyze how participation in SNSs affects human relations. The findings of this strand of the literature support the hypothesis that online interactions play a role in the preservation and development of social ties against the threats posed by the weakening of community life and the erosion of the stock of social capital.

SNSs have been claimed to support the strengthening of bonding and bridging social capital (Lee, 2013; Steinfield, Ellison, & Lampe, 2008) and the social integration and well-being of the elderly (Näsi, Räsänen, & Sarpila, 2012; Russel, Cambell, & Hughes, 2008), to allow the crystallization of weak or latent ties that might otherwise remain ephemeral (Ellison, Steinfield, & Lampe, 2007; Haythornthwaite, 2005), to help users to cope with social anxiety and negative moods associated with loneliness (Clayton, Osborne, Miller, & Oberle, 2013; Grieve, Indian, Witteveen, Tolan, & Marrington, 2013), to support teenagers' self-esteem, encouraging them to relate to their peers (Ellison et al., 2011; Trepte & Reinecke, 2013), and to promote civic engagement and political participation (Gil de Zúñiga, Jung, & Vanenzuela, 2012; Zhang, Johnson, Seltzer, & Bichard, 2010[6]).

Drawing on survey data from a random sample of 800 undergraduate students, Ellison et al. (2007) find that certain types of Facebook use can help individuals accumulate and maintain bridging social capital. Their results support the hypothesis that the social network helps students to overcome the barriers to participation so that individuals who might otherwise shy away from initiating communication with others are encouraged to do so through the Facebook infrastructure. In the authors' words, highly engaged users are using Facebook to ''crystallize'' relationships that might otherwise remain ephemeral.

Steinfield et al. (2008) analyzed panel data from two surveys on Facebook users conducted a year apart at a large U.S. university. Intensity of Facebook use in year one strongly predicted bridging social capital outcomes in year two, even after controlling for measures of self-esteem and satisfaction with life. The authors suggest that Facebook affordances help reduce barriers that students with lower self-esteem might experience in forming the kinds of large, heterogeneous networks that are sources of bridging social capital. However, the literature on Facebook suggests that the social network and, more generally, Internet-mediated communication serves more to preserve relations among offline contacts than to activate latent ties or create connections with strangers (Ellison et al., 2007; Pénard & Poussing, 2010). In one of the rare economic studies on the topic, Pénard and Poussing (2010) draw on data from the 2002 wave of the European Social Survey from Luxembourg to find that people who already have a large stock of social capital are more likely to use the Internet to foster social relationships. In a recent paper based on data drawn from the 2008 section of the German Socio-Economic Panel and confidential data provided by *Deutsche Telekom*, Bauernschuster, Falck, and Woessmann (2014) find that having broadband Internet access at home has positive effects on an individual's frequency of visiting theatres, the opera and exhibitions, and on the frequency of visiting friends. Exploring a subsample of children aged 7–16 living in the sampled households, the authors further find evidence that having broadband Internet access at home increases the number of children's out-of-school social activities, such as playing sports, taking ballet, music or painting lessons, or joining a youth club.

[6]It is worth noting that part of the literature does not agree with the above reported claims about the beneficial effects of Internet-mediated interaction on social capital. Early sociological studies on computer-mediated communication shared the fear that the Internet would cause a progressive reduction in social interactions. The main argument shared by Internet skeptics was based on the presumption that the more time people spend using the Internet during leisure time, the more time is detracted from social activities (Gershuny, 2003; Katz, Rice, & Apsden, 2001; Kraut et al., 1998; Nie & Erbring, 2000; Nie, Sunshine Hillygusm, & Erbring, 2002). Studies emphasizing the possibly negative correlation between Internet usage and sociability date back to just shortly before the explosion of online networking, which, in our view, has made their results anachronistic. Today, sceptic authors warn that, beyond a certain threshold, the development of human relationships by the exclusive means of online interactions may prevent users from enjoying those emotional benefits normally associated with face-to-face interactions (see, e.g., Lee, 2013). Based on Italian survey data, Sabatini and Sarracino (2014b) found a significantly negative association between the use of SNSs and subjective well-being, probably mediated by a detrimental effect of SNSs' use on social trust. A survey of the literature accurately describing the different positions on the role of Internet-mediated interaction in the accumulation of social capital is included in Antoci et al. (2013).

More recently, Sabatini and Sarracino (2014a) used data drawn from the Italian Multipurpose Household Survey to analyze the impact of participation in social networking sites on offline social participation. The authors find that online participation is significantly and positively associated with the frequency of face-to-face meetings with friends and acquaintances.

In this article, we suggest a theoretical framework for analyzing the complementary role of offline (or face-to-face) and online interactions in the evolution of social participation implicitly suggested by the above-mentioned studies. Our contribution to this on-going debate consists in the development of an evolutionary game model analyzing how human relations may evolve in a context potentially characterized by declining face-to-face interactions and growing online social participation.

3. THE EVOLUTIONARY GAME MODEL

Let us assume that, in each period of time t, individuals play a one-shot population game (i.e., all agents play the game simultaneously) and that they have to choose (*ex ante*) one of the strategies we briefly introduced above:

1. "No social participation" (*NSP*),
2. "Social participation with FF-interactions" (*SPF*), or
3. "Social participation with SN-interactions" (*SPN*).

As explained in the introduction, the *NSP* strategy entails withdrawal from every unnecessary social interaction. *NSP* agents devote all their time to work and/or to forms of private consumption, that is, the consumption of goods that can be enjoyed alone. More specifically, following Antoci, Sabatini, and Sodini (2012b), we assume that material products may replace relational goods, or at least may be used to compensate for the lack of human interaction, when the social environment has few opportunities for participation.

The *SPF* strategy entails only physical encounters and excludes any Internet-mediated interaction.

With the *SPN* strategy, on the other hand, individuals participate socially through both online interactions and face-to-face encounters. In the *SPN* way of living, online interactions result in a certain degree of physical encounters as a by-product. This amount may be limited; in this case, the social participation of individuals choosing *SPN* basically consists of online contacts with peers they will never meet in person. Or, by contrast, *SPN* agents may use online communication exclusively as a means to improve their face-to-face sociability. The empirical literature summarized in Section 2 shows that, in most cases, instant messaging, and interactions through social networks, emails and other kinds of Internet-mediated communication are preparative and instrumental to physical interactions (Ellison et al., 2007, 2011; Gil de Zúñiga et al., 2012; Steinfield et al., 2008). In the latter case, it would seem reasonable to assume a configuration of payoffs allowing the *SPN* strategy to Pareto-dominate on *SPF*. The share of time that *SPN* agents devote to physical interactions may also vary according to the characteristics of communities, reference groups, and material needs. For example, if most fellow community members participate in online networks, then online contacts are more likely to be instrumental to actual interactions (Ellison et al., 2007, 2011; Hampton & Wellman, 2003). Drawing on survey and ethnographic data from a wired suburb of Toronto, Hampton and Wellman (2003) show how high-speed, always-on access to the Internet, coupled with a local online discussion group, transformed and enhanced "neighboring" through the increase in physical contacts with weaker ties, the improvement of strong ties, and the promotion of discussions and mobilization around local issues. Reference groups also play a crucial role in determining the extent to which online interactions entail physical encounters as a by-product. Online communities may regularly arrange physical meetings, as generally happens in hobby networks, such as those of photographers connected through Flickr or specialized forums (Valenzuela, Park, & Kee, 2009). On the other hand, reference groups may act as a deterrent to physical encounters, as in the case of *hikikomoris*, who avoid face-to-face interactions and develop their entire social lives online (Furlong, 2008; Kato et al., 2012). Time and distance constraints also determine the extent to which online interactions generate physical encounters. According to the computer-mediated communication literature, online networks play a vital role in the preservation of social ties despite scheduling problems and distance and mismatches in the time available for leisure. In fact, online social participation favors asynchronous interactions, which allow individuals to compensate for a lack of time; one can benefit from the others' participation, for example, by reading and replying to a message or seeing a photo or a note and commenting on it, even if the person who wrote the message or shared the content is currently offline (Antoci et al., 2013).

There are a number of reasons to distinguish *SPF* from *SPN* types of social participation and to consider both of them in our model (instead of only accounting for the *SPN* strategy). First, the *SPN* strategy entails positive activation costs; a device and access to the Internet are required for computer-mediated interaction. Residents in areas with no access to broadband Internet due, for example, to the lack of proper infrastructures may be particularly disadvantaged. In addition, participation through online networks requires specific relational skills. For example, one needs to update her/his Facebook account regularly, and to reply to comments and messages. The activation costs are particularly high for specific groups, such as the elderly or economically disadvantaged individuals, who may not even consider the possibility of computer-mediated communication. As descriptive statistics clearly show, there is still a portion of the population who never use the Internet (Duggan & Brenner, 2013).

In addition, the distinction between the two strategies allows us to point out that, even if the hypothetical scenario where all agents adopt the *SPF* strategy may be Pareto-superior, the possibility of playing the *SPN* strategy can lead to a second-best scenario, allowing society to avoid social poverty traps (i.e., those situations where all agents play *NSP*), as the dynamics described in this article clearly show (see Section 4). From this perspective, the *SPN* strategy can be seen as a way to defend social interaction, which in respect to the *SPF* way of participation, has the advantage of being less vulnerable to the negative social externalities caused by any increase in a share of the population adopting *NSP*. In addition, Internet-mediated communication fosters coordination, making the *SPN* strategy more "efficient." More in general, both the *NSP* and the *SPN* strategies can be seen as defensive choices through which individuals try to cope with deterioration in the social environment, for example, in terms of social participation opportunities. However, the *SPN* strategy creates positive social externalities while the *NSP* does not (or, worse, it may create negative social externalities). The case of *hikikomoris* can be used once again to illustrate this argument. In a world without the Internet, where the *SPN* strategy does not exist, *hikikomori* individuals are doomed to withdraw from social life. They become part of the *NSP* share of the population and their unavailability for social life negatively influences the sociability of others. With Internet-mediated interactions, i.e., the possibility of playing the *SPN* strategy, socially withdrawn individuals are at least available for online relationships and may positively contribute to the relational life of others. It can be argued that individuals who develop their entire social life online are likely to suffer from declining well-being and health, as studies on the detrimental effects of Facebook use suggest (Chou & Edge, 2012; Forest & Wood, 2012; Kim, LaRose, & Peng, 2009; Kross et al., 2013). However, a complete retirement from any form of social life is doomed to produce even worse and irreversible effects, as demonstrated by *hikikomoris* before the growth of online networks.

We represent the population of individuals with the vector $x=(x_1, x_2, x_3)\in R^3$, where x_1, x_2 and x_3 indicate the shares of individuals choosing strategies *NSP, SPF* and *SPN*, respectively. Thus $x_i \geq 0$, all i, and $\sum_i x_i = 1$; so x belongs to the 2-dimensional simplex S (see Figure 1).

For simplicity, we assume that the payoff functions are linear in the variables x_1, x_2 and x_3 :

$$\begin{aligned}
\Pi_{NSP} &= \alpha \\
\Pi_{SPF} &= (\beta-\gamma)x_1 + (\beta+\delta)x_2 + \beta x_3 \\
\Pi_{SPN} &= (\varepsilon-\eta)x_1 + \varepsilon x_2 + (\varepsilon+\lambda)x_3
\end{aligned} \tag{1}$$

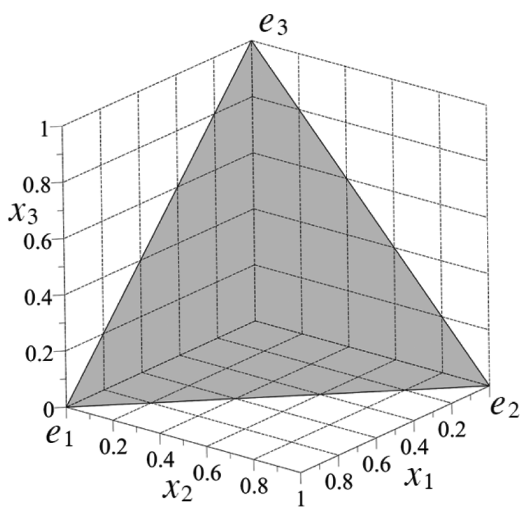

FIGURE 1 The 2-dimensional simplex S.

where $\alpha, \beta, \gamma, \delta, \varepsilon, \eta, \lambda$ are strictly positive parameters, therefore $\varepsilon - \eta < \varepsilon < \varepsilon + \lambda$ and $\beta - \gamma < \beta < \beta + \delta$. Matrix (2) represents the payoffs obtained by the strategies *NSP, SPF*, and *SPN* when they are played in "pure" population contexts in which all individuals are adopting only one strategy (either *NSP* or *SPF* or *SPN*):

$$\begin{array}{llll} & \textit{all play NSP} & \textit{all play SPF} & \textit{all play SPN} \\ NSP & \alpha & \alpha & \alpha \\ SPF & \beta - \gamma & \beta + \delta & \beta \\ SPN & \varepsilon - \eta & \varepsilon & \varepsilon + \lambda \end{array} \tag{2}$$

Notice that the parameters β and ε are pivotal. The parameter β measures the payoff of an individual playing the *SPF* strategy "against" a population entirely composed of *SPN* players, while the parameter ε represents the payoff of an individual playing the *SPN* strategy and interacting with a population entirely composed of *SPF* players.

According to the payoff functions (1), the increase in the share x_1 of the *NSP* population is always undesirable (being $\varepsilon - \eta < \varepsilon < \varepsilon + \lambda$ and $\beta - \gamma < \beta < \beta + \delta$). By contrast, any increase in x_2 (x_3 being constant) or in x_3 (x_2 being constant) always improves the payoffs of both *SPF* and *SPN*.

Notice that *SPN*'s payoffs benefit more from increases in x_3 (the share of the population playing *SPN*) than from increases in x_2, and *SPF*'s payoffs benefit more from an increase in x_2 (the share of the population playing *SPF*) than in x_3. On the other hand, if x_1 is constant, an increase in x_3 entails a reduction in the payoff of *SPF* and, vice versa, increases in x_2 entail a reduction in the payoff of *SPN*.

According to the payoff functions (1) and matrix (2), we have that:

1. The *NSP* strategy strictly dominates the others if $\alpha > \beta + \delta, \varepsilon + \lambda$.
2. The *SPF* strategy strictly dominates the others if $\beta - \gamma > \alpha, \varepsilon + \lambda$.
3. The *SPN* strategy strictly dominates the others if $\varepsilon - \eta > \alpha, \beta + \delta$.
4. The pure population state $(x_1, x_2, x_3) = (1, 0, 0)$, in which all individuals adopt the *NSP* strategy, is a (strict) Nash equilibrium if $\alpha > \beta - \gamma, \varepsilon - \eta$.
5. The pure population state $(x_1, x_2, x_3) = (0, 1, 0)$, in which all individuals adopt the *SPF* strategy, is a Nash equilibrium if $\beta + \delta > \alpha, \varepsilon$.
6. The pure population state $(x_1, x_2, x_3) = (0, 0, 1)$, in which all individuals adopt the *SPN* strategy, is a Nash equilibrium if $\varepsilon + \lambda > \alpha, \beta$.

Notice that the pure population states $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ can be simultaneously Nash equilibria. In this work we assume that the equilibrium selection process is driven by the well-known replicator dynamics; according to these dynamics, the pure population states, which are Nash equilibria, are also locally attractive stationary states. This property also holds under any other payoff monotonic selection process (see Weibull, 1995). To focus on the more relevant dynamic regimes, we shall analyze replicator equations under the following assumptions.

Assumption 1 ($\varepsilon - \eta > \beta - \gamma$). If all individuals adopt *NSP* ($x_1 = 1$ and $x_2 = x_3 = 0$), the payoff of *SPN* is higher than the payoff of *SPF*. In our framework, *SPN* is, in fact, a more rewarding response (than *SPF*) to the impoverishment of the social environment for at least two reasons. First, *SPN* players can benefit from the "global" stock of social capital accumulated worldwide on the Internet. This stock (K_S) is a public good, in that it potentially benefits whoever is connected to the Web and adopts the *SPN* strategy. K_S allows instantaneous or asynchronous interactions with people who are too distant, or who have different working hours, to physically interact with our population. It is worth noting that asynchronous interactions may further help people to reconcile constraints related to mobility with the need to take care of social relationships; when individuals cannot meet in person, or conduct long-distance chats due to time differences, the social capital of the Internet offers the possibility of a quality, though deferred, interaction. Second, the use of Internet-mediated interaction by *SPN* may allow a more efficient use of time in that it entails a better coordination among agents and a higher diffusion of information on how to use leisure time. These mechanisms do not hold for *SPF* players because, in a limited though open population where everyone plays *NSP* and there is no "physical" participation, individuals cannot benefit from any stock of social capital (which, following Bourdieu [1980] exists only insofar as it is shared).

In our framework, the stock of social capital is exogenously given. In fact, K_S is accumulated because of the social participation choices of all individuals using Internet-mediated interaction worldwide, and cannot be influenced—not in the short run anyway—by the strategies adopted by the population in our framework. In addition, it seems reasonable to assume that K_S has a lower velocity of variation in respect to the shares x_1, x_2 and x_3 of the population adopting the three alternative strategies.

Assumption 2 ($\alpha > \beta - \gamma$). This assumption means that (see the payoff matrix (2)), if all individuals adopt *NSP*, the payoff of *NSP* is higher than that of *SPF* (the payoff of *SPN* is described above in Assumption 1). This and the following assumptions allow us to limit the analysis to the more interesting cases where the state ($x_1 = 1$ and $x_2 = x_3 = 0$) is a social poverty trap which, without the possibility of choosing the *SPN* strategy, would always be attractive.

Assumption 3 ($\beta < \varepsilon + \lambda$). If the entire population adopts *SPN* (i.e., $x_3 = 1$ and $x_1 = x_2 = 0$), the payoff of this strategy is higher than that of *SPF*. In a world where everyone participates both through online networks and face-to-face encounters, not being online is necessarily less rewarding than joining the networks. On the other hand, being outside of the network (i.e., playing the *SPF* strategy) implies an increasing relational cost. Consider, for example, an *SPF*-playing teenager whose classmates join Facebook. Not following them into the network may lead to the cooling of some relationships as well as to exclusion from those established through the activation of latent ties via Internet-mediated interaction.

Assumption 4 ($\alpha < \beta + \delta$). As a consequence, the payoff of the *NSP* strategy when all individuals adopt *NSP* ($x_1 = 1$ and $x_2 = x_3 = 0$) is lower than the payoff of the *SPF* strategy when all individuals adopt *SPF* ($x_2 = 1$ and $x_1 = x_3 = 0$).

Assumption 5 ($\alpha < \varepsilon + \lambda$). The payoff of the *NSP* strategy when all individuals adopt *NSP* ($x_1 = 1$ and $x_2 = x_3 = 0$) is lower than the payoff of the *SPN* strategy when all individuals adopt *SPN* ($x_3 = 1$ and $x_1 = x_2 = 0$).

According to matrix (2), it is more rewarding for a *SPF* player to interact with a population of *SPN* players (payoff: β) than with a population of *NSP* players (payoff: $\beta - \gamma$); however, playing the *SPF* strategy within a population of *SPF* players provides the highest possible payoff ($\beta + \delta$). Similarly, a *SPN* player also receives the highest possible payoff when she meets a population only composed of *SPN* players. If the *SPN* strategy spreads at the expense of withdrawal from social participation (*NSP* strategy), then its diffusion benefits all individuals who play *SPN* or *SPF* (with higher benefits for the former players).

Assumptions 4 and 5, entailing that the state where all individuals adopt *NSP* is Pareto-dominated both by the state where everyone plays *SPF* and by the state where the entire population plays *SPN*, serve to restrict the analysis to the dynamics where the scenario ($x_1 = 1$ and $x_2 = x_3 = 0$) is always a "social poverty trap" (Antoci et al., 2007). This allows us to focus on the main objective of the model, that is, to analyze how online networking may work as a "defensive choice" that reduces the risk of falling into the social poverty trap caused by a decline in social participation.

The payoff of the *NSP* strategy is constant (and equal to α) and not subject to externalities. This strategy can thus be seen as a defensive behavior aimed at protecting individuals from the negative consequences of a decline in social participation. As explained in section 2, the same "protecting" role is played by the *SPN* way of participation. However, while *NSP* creates negative externalities for those who play alternative strategies, *SPN* creates positive externalities for *SPN* and, to a lesser extent, also for *SPF* players. *SPF* and *SPN* strategies, however, generate positive externalities only insofar as the increase in the respective shares of the population x_2 and x_3 is associated with a reduction in x_1. If x_1 is constant, then any increase in x_2 (x_3) generates negative externalities for *SPN* (*SPF*) players.

We are aware that, in principle, the payoff of *NSP* may negatively depend on x_3 and x_2, for two reasons: (a) individuals playing *NSP* may be envious of those who work less and enjoy a social environment rich of participation opportunities, and (b) the adoption of *NSP* creates positive externalities on production.

Our choice not to account for these relationships in the model is due to our focus on the self-feeding mechanisms with which each strategy spreads among the population. In fact, each payoff grows as much as the related strategy spreads to the detriment of the other two. The assumption that the payoff of *NSP* negatively depends on x_2 and x_3 would not change the self-feeding nature of the diffusion mechanisms and its consequences in terms of welfare.

In our evolutionary game model, at each instant of time t, the distribution of strategies $[x_1(t), x_2(t), x_3(t)]$ determines their relative performances, which drives social evolution in the sense that the strategies that turn out to be more rewarding are imitated and, by replicating faster, manage to proliferate at the expense of the less rewarding ones. Time is continuous and the population is modelled as a continuum of players. Following Taylor and Jonker (1978), we assume that the growth rates $\dot{x}_i/x_i = (dx_i/dt)/x_i$ of the shares x_i, $i = 1, 2, 3$, are given by the well-known replicator equations (also see Weibull, 1995):

$$\begin{aligned} \dot{x}_1 &= x_1\left(\Pi_{NSP} - \overline{\Pi}\right) \\ \dot{x}_2 &= x_2\left(\Pi_{SPF} - \overline{\Pi}\right) \\ \dot{x}_3 &= x_3\left(\Pi_{SPN} - \overline{\Pi}\right) \end{aligned} \tag{3}$$

where $\overline{\Pi}$ represents the population-wide average payoff:

$$\overline{\Pi} = x_1 \cdot \Pi_{NSP} + x_2 \cdot \Pi_{SPF} + x_3 \cdot \Pi_{SPN}$$

The dynamic system (3) is analyzed in the Appendix using Bomze's (1983) classification for replicator equations. In the following section, we illustrate the basic features of dynamics generated by (3).

4. SCHEDULING PROBLEMS

Winship (2009) shows how the adoption of a non-participation strategy (the *NSP* strategy in this article, the "reading strategy" in his article) can be fueled by *scheduling* problems. Let us consider, for example, the following context. Individuals interact in occasion of random pairwise encounters. Each period of time t is divided into two subperiods t_1 and t_2 of equal magnitude. Each individual has to choose (*ex ante*) one of the following strategies:

A: to work in both periods t_1 and t_2 (a nonparticipation strategy),
B: to work only in t_1 and to devote t_2 to social participation, or
C: to work only in t_2 and to devote t_1 to social participation.

We assume that the payoffs of pairwise encounters are given by the following matrix:

$$\begin{array}{cccc} & A & B & C \\ A & \alpha & \alpha & \alpha \\ B & 0 & \beta & 0 \\ C & 0 & 0 & \beta \end{array}$$

with $\alpha, \beta > 0$. According to this payoff matrix, the A strategy is a self-protection strategy (as the strategy *NSP* described above or the reading strategy considered in Winship, 2009) in that it allows individuals to get the payoff α whatever the choice of the opponent player is. Differently from the A strategy, strategies B and C give a positive payoff (β) only if the interactions take place between individuals adopting the same time allocation choice. In this context, indicating the shares of individuals choosing the A, the B and the C strategy with x_1, x_2 and x_3, we have that:

a. If $\beta > \alpha$ holds, then three Nash equilibria exist, $(x_1, x_2, x_3) = (1, 0, 0)$, $(0, 1, 0), (0, 0, 1)$, where all individuals play, respectively, the A, the B, and the C strategy; moreover, the equilibrium (1,0,0) is Pareto-dominated by the others.

b. If $\beta < \alpha$, then the A-strategy strictly dominates the B and the C strategies and a unique Nash equilibrium exists: (1,0,0).

In the payoff context (a) we have that, under any payoff monotonic dynamics (see Weibull, 1995), all the states $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ are locally attractive stationary states; in the context (b), the state (1,0,0) is globally attractive. The local attractivity of the states $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ in the context (a) implies path dependent dynamics: equilibrium selection depends on the initial distribution of strategies (the corresponding dynamic regime is similar to that illustrated in Figure 5).

As illustrated in Winship (2009), the complexity of the choice processes increases when individuals have to choose not only how to allocate their time between social interaction and work, but also between different types of social interaction (e.g., if they have to decide whether to go either to cinema or to the theatre). This more complex scenario may give rise to coordination problems similar to those arising in "battle of sexes" contexts (see, among others, Antoci, Galeotti & Sacco, 2000).

Relative to face-to-face encounters, Internet-mediated interaction can help overcoming scheduling problems in two main ways. First, if a face-to-face meeting is not possible at time t due to a scheduling problem (e.g., because of different working times), *SPN* players can interact asynchronously. This may be less gratifying but still enjoyable and effective. Second, the *SPN* strategy allows players to better match with individuals with the same time allocation choice and the same preferences.

In this article, we have chosen not to explicitly model this second aspect. Rather, we focus on the role of asynchronous interactions. As illustrated in section 3, *SPN* players can exploit the stock of the "Internet social capital," K_S, to interact with other *SPN* players when they are offline. Internet-mediated interactions do not have to take place in real time: one can comment on a thread on Facebook or reply to a private message even when the initiator of the thread or the sender is offline. As suggested in the applied psychology literature, asynchronous interactions play a fundamental role in the preservation of social ties against the threats of busyness and mobility (Ellison et al., 2007). This may be the case, for example, of a *SPN* player who works the night shift and wishes to communicate with another who works the day shift, or of migrant workers who moved to a different time zone and want to stay in touch with family and friends in their country of origin.

5. CLASSIFICATION OF DYNAMICS

In this section, we describe the possible dynamic regimes that can be observed under dynamics (3), defined in the 2-dimensional simplex (see Figure 1):

$$S = \left\{ (x_1, x_2, x_3) : x_i \geq 0, i = 1, 2, 3, \sum_i x_i = 1 \right\}$$

These regimes are illustrated in Figures 2–5. In these figures, the vertices $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$ represent the points where, respectively, only the strategy *NSP, SPF* and *SPN* is adopted. An *edge* of the simplex S consists of all population states in which a given (fixed) strategy is not adopted; we shall denote by $e_i - e_j$ the edge joining e_i with e_j. Thus, the edges $e_1 - e_2$, $e_1 - e_3$ and $e_2 - e_3$ are the edges where only strategies *NSP-SPF, NSP-SPN,* and *SPF-SPN* are, respectively, adopted in the population of individuals.

The vertices e_1, e_2 and e_3 are always fixed points under replicator dynamics (3). The other fixed points of replicator dynamics are the states in which all the strategies

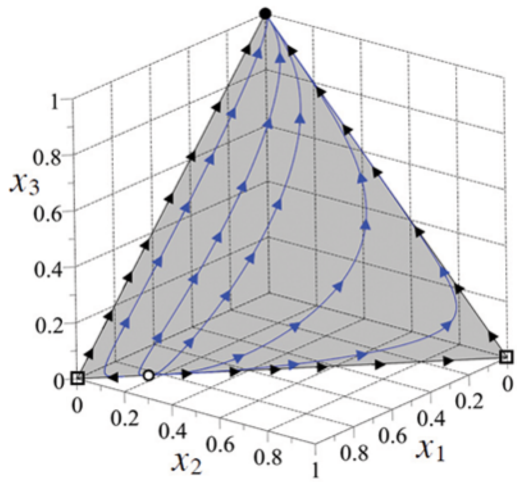

FIGURE 2 Case 1: The vertex e_3, where all individuals adopt the strategy *SPN*, is globally attractive in the interior of the simplex *S*.

played by a strictly positive share of individuals give the same payoffs (see Weibull, 1995). In Figures 2–5, attractive fixed points are indicated by full dots, repulsive ones by open dots and saddle points by squares.

The analysis contained in the Appendix allows us to provide the following classification of possible dynamic regimes under dynamics (3):

Case 1 ($d = \varepsilon - \alpha - \eta \geq 0$ and $e - b = \varepsilon - \beta - \delta \geq 0$). In this case, the vertex e_3, where all individuals adopt the strategy *SPN*, is globally attractive in the interior of the simplex *S* (see Figure 2); that is, every trajectory starting from a point in the

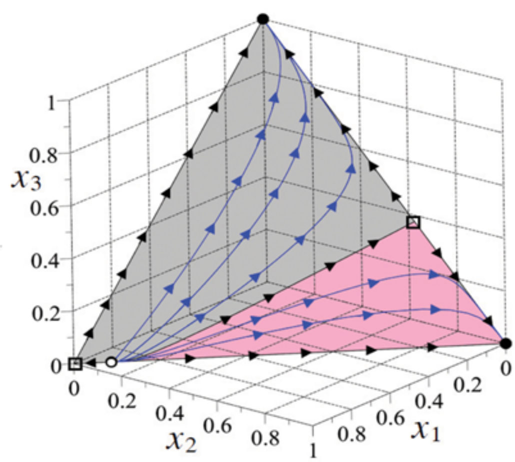

FIGURE 3 Case 2: The vertices e_2 and e_3, where all players adopt the strategies *SPF* and *SPN*, respectively, are locally attractive.

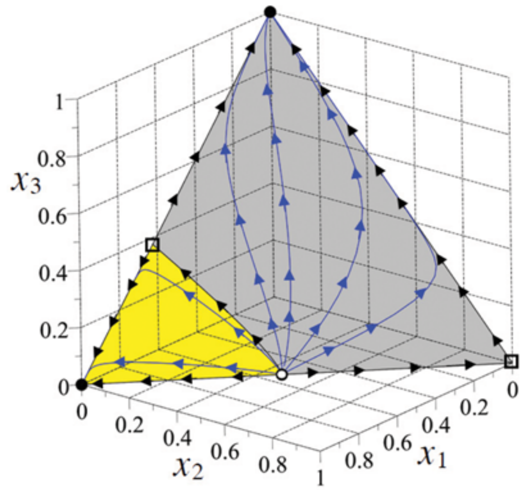

FIGURE 4 Case 3: The vertices e_1 and e_3, where all players adopt the strategies *NSP* and *SPN*, respectively, are locally attractive.

interior of S approaches e_3. Notice that on the edge $e_1 - e_2$, where no one plays the *SPN* strategy, a bi-stable dynamic regime occurs: the stationary states e_1 and e_2 are locally attractive and their basins of attraction are separated by the repelling stationary state (indicated by the open dot) lying in the interior of the edge $e_1 - e_2$. Consequently, society collapses in e_1, where everyone withdraws from social participation, if the initial proportion of *NSP* players is high enough (and no one plays the *SPN* strategy). This means that, without the option of playing *SPN*—that is, in the absence of online networks—the vertex e_1 acts as a social poverty trap, where the

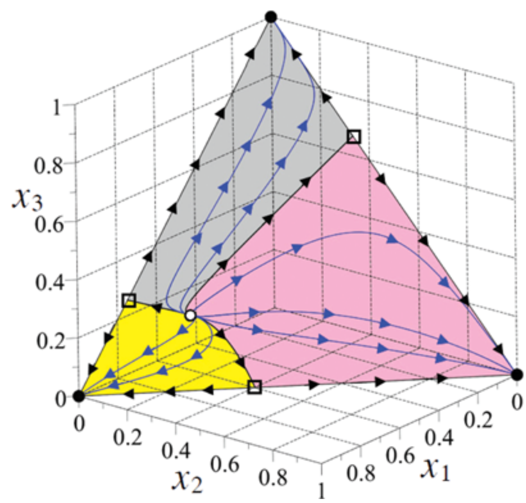

FIGURE 5 Case 4: All the fixed points in the vertices e_1, e_2 and e_3 are locally attractive.

payoff of the entire population is, by assumption, lower than in e_2 (where the whole population plays *SPF*). The introduction of online networking (i.e., of the *SPN* strategy) then makes e_1 a saddle point. The *SPN* strategy plays the same role in preventing society from falling into a social poverty trap in Case 2, described below.

Case 2 ($d = \varepsilon - \alpha - \eta \geq 0$ and $e - b = \varepsilon - \beta - \delta < 0$). In this case, we obtain a bi-stable dynamics regime; that is, the vertices e_2 and e_3 (where all players adopt the strategies *SPF* and *SPN*, respectively) are locally attractive, while the other fixed points are repulsive or saddles. The basins of attraction of e_2 and e_3 are separated by the stable branch of the saddle point lying in the edge $e_2 - e_3$ (see Figure 3).

Case 3 ($d = \varepsilon - \alpha - \eta < 0$ and $e - b = \varepsilon - \beta - \delta \geq 0$). In this case, we also obtain a bi-stable dynamics regime. However, the attractive vertices are e_1 and e_3 (where all players adopt the strategies *NSP* and *SPN*, respectively); all the other fixed points are repulsive or saddles. According to Bomze's classification, this parameter configuration is compatible with three of the phase portraits—9, 37, and 38—shown in Bomze's paper. These phase portraits possess the common feature according to which "almost all"[7] the trajectories of system (3) approach either e_1 or e_3. Figure 4 represents the phase portrait number 9; in this figure, the basins of attraction of e_1 and e_3 are separated by the stable branch of the saddle point lying in the edge $e_1 - e_3$.

Case 4 ($d = \varepsilon - \alpha - \eta < 0$ and $e - b = \varepsilon - \beta - \delta < 0$). In this case, all the fixed points in the vertices e_1, e_2 and e_3 are locally attractive; such parameters configuration is compatible with only two of the phase portraits illustrated in Bomze's paper—7 and 35. In these phase portraits, "almost all" (in the sense of footnote 11) trajectories approach the vertices e_1, e_2 and e_3. Figure 5 shows the phase portrait number 7, where there exists a repulsive fixed point in the interior of S and the fixed points in the edges are saddle points whose stable branches separate the basins of attraction of e_1, e_2 and e_3.

6. WELL-BEING

Matrix (2) indicates that: (a) in the vertex e_1, where everyone plays the *NSP* strategy ($x_1 = 1$ and $x_2 = x_3 = 0$), the *NSP*'s payoff is α ; (b) in the vertex e_2, where everyone plays the *SPF* strategy ($x_2 = 1$ and $x_1 = x_3 = 0$), the *SPF*'s payoff is $\beta + \delta$; and (c) in the vertex e_3, where everyone plays the *SPN* strategy ($x_3 = 1$ and $x_1 = x_2 = 0$), the *SPN*'s payoff is $\varepsilon + \lambda$.

Due to Assumptions 4 and 5, we have $\alpha < \beta + \delta$ and $\alpha < \varepsilon + \lambda$; the state e_1 where everyone plays *NSP* is Pareto-dominated by both e_2 and e_3.

The analysis of dynamics has shown that e_1 can be locally attractive—that is, it may be a social poverty trap—even if there is the possibility of developing human interactions online through the *SPN* strategy. However, if the *SPN* strategy is rewarding enough in respect to *NSP* (i.e., $\varepsilon - \alpha - \eta \geq 0$), then e_1 ceases being a trap, in that it becomes a saddle point.

[7]That is, excluding the trajectories coinciding with the other fixed points or those belonging to the stable branches of saddles.

The state e_2, where everyone plays *SPF*, Pareto-dominates e_3, where everyone plays *SPN*, if $\beta + \delta > \varepsilon + \lambda$. If this is the case, it can be easily shown that vertex e_2 is locally attractive. This means that, in the dynamics described in Cases 1 and 3 (see Figures 2 and 4) of the previous classification, e_3 is never Pareto-dominated by e_2. However, in Cases 2 and 4 (see Figures 3 and 5), e_2 can Pareto-dominate e_3 or vice versa, even if both states are locally attractive.

In the former case, e_3 can be intended as a second-best equilibrium which, anyway, strictly Pareto-dominates e_1. In this scenario, the *SPN* strategy can prevent society from falling into the social poverty trap, but this entails convergence to state e_3, where all agents play *SPN,* which is Pareto-dominated by e_2 (where all agents play *SPF).* The configuration of parameters in the simplex's vertexes assumed in this case reflects the claims emerging from the literature reviewed in section 2: Internet-mediated interactions play a crucial role in preventing the disruption of ties against the threats posed by mobility, sprawl, the reduction in leisure time and the decline in social participation. These interactions do not substitute for physical encounters; rather, they are complements (Cuberes, 2013). From this point of view, a state where everyone plays *SPF* could be equivalent, in terms of well-being, to one where all agents play *SPN*. However, in contrast to face-to-face interactions, online interactions may entail a number of negative consequences for individual well-being, which suggests that a state where all agents play *SPF* may be Pareto-superior. That said, it must be remembered that the possibility of interacting online, that is, to play *SPN*, wards off the worst possible state where all agents withdraw from social participation, that is, play *NSP*, and society falls into a social poverty trap.

The latter case, where e_2 and e_3 are both locally attractive and e_3 Pareto-dominates e_2, accounts for the more optimistic views on the effect of online interactions on social capital and well-being, and for the hypothesis that social relationships exclusively based on physical encounters may also produce negative externalities in the sense of inhibiting the diffusion of the Internet and social networks. In communities where most individuals prefer to avoid computer-mediated interactions, the diffusion of the Internet may in fact be discouraged and the risk of falling into a social poverty trap may be higher.

7. CONCLUSIONS

In this article, we draw on the empirical literature on social capital and computer-mediated communication to develop a theoretical framework analyzing how human relations may evolve in a context characterized by the decline in face-to-face social participation documented by empirical studies and the rapid growth of online social participation that we have witnessed in recent years.

We assume that individuals can react to increasing busyness and the reduction in leisure time by withdrawing from social participation, or by developing a part of their social relationships online in order to overcome the barriers of distance and time. As in Antoci et al. (2012a, 2013), interpersonal interactions can be developed through two alternative forms of social participation: (a) a social networking strategy, *SPN*, where participation takes place both by means of online networking and face-to-face encounters, and (b) a face-to-face strategy, *SPF*, which is entirely developed through in-person encounters and entails no Internet-mediated interaction. In addition to the hypotheses advanced in Antoci et al. (2012a, 2013), agents can choose to withdraw from social participation and to devote all their time to work and private consumption (*NSP* strategy).

In a world where the only alternative to face-to-face interaction is withdrawal from participation, the state where nobody participates (i.e., all agents play *NSP*) is attractive, but constitutes a social poverty trap. The possibility of interacting online, on the other hand, offers an effective coping response, allowing individuals to "defend" their social life from increasing busyness and the reduction in leisure time, leading society to a state where all agents develop social interactions through a mixed strategy encompassing, to diverse extents, both physical encounters and involvement in online networks. In some cases, this is a second-best scenario that is Pareto-dominated by the state where all agents participate only through physical encounters (i.e., by playing *SPF*). However, the possibility of playing *SPN* allows society to avoid the (otherwise attractive) worst-case scenario where nobody participates.

This article represents a new step in a research program aimed at analyzing the evolution of social participation and the accumulation of social capital in relation to economic growth and technological progress. In previous works, we highlighted how the reduction in the time available for social participation can trigger self-feeding processes leading to the progressive erosion of the stock of social capital (Antoci et al., 2012b, 2013). There, we analyzed a scenario in which the time for social participation is an endogenous variable (i.e., it depends on agents' allocation choices), social relationships can be developed only by means of face-to-face interaction and agents may only react to increasing busyness by replacing participation with private activities—or, in other words, to replace the production and consumption of relational goods with the production and consumption of private goods. In the present article, we address a scenario in which agents can also interact with each other through both actual encounters and online networking. A number of relevant research questions remain unanswered and are worthy of further investigation. In our analysis, we do not advance assumptions about how face-to-face and computer-mediated interactions may be distributed within the *SPN* strategy

The role of online networks in the development of interpersonal relationships and in the preservation of social cohesion against the threat of social poverty traps suggest that individuals and communities who do not have access to the Internet—due, for example, to the absence of proper infrastructures such as broadband, or to lack of the skills required to participate in SNSs—may increasingly suffer from difficulties in social integration. In this scenario, the digital divide is likely to become an increasingly important factor of social exclusion, which may significantly exacerbate inequalities in well-being and capabilities.

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APPENDIX: BASIC MATHEMATICAL RESULTS

Notice that posing:

$$A = \begin{pmatrix} \alpha & \alpha & \alpha \\ \beta - \gamma & \beta + \delta & \beta \\ \varepsilon - \eta & \varepsilon & \varepsilon + \lambda \end{pmatrix}.$$

The replicator Eq. (3) can be written in the following form:

$$\dot{x}_i = x_i(e_i \cdot Ax - x \cdot Ax), i = 1, 2, 3,$$

where $x = (x_1, x_2, x_3)$ while $e_i, i = 1, 2, 3$, represent the canonical basis $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$.

We analyze dynamics (3) by using Bomze's (1983) classification for replicator equations. In order to use Bomze's classification, we need to rewrite the payoff matrix A in the following form:

$$B = \begin{pmatrix} 0 & 0 & 0 \\ a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ \beta - \gamma - \alpha & \beta + \delta - \alpha & \beta - \alpha \\ \varepsilon - \eta - \alpha & \varepsilon - \alpha & \varepsilon + \lambda - \alpha \end{pmatrix}, \qquad (4)$$

with the first row made of zeros.[8]

Remember that, according to Assumptions 1–5, $\alpha, \beta, \gamma, \delta, \varepsilon, \eta, \lambda$. are strictly positive parameters satisfying the following conditions:

$$\gamma > \eta, \varepsilon - \eta > \beta - \gamma, \alpha > \beta - \gamma, \beta < \varepsilon + \lambda, \alpha < \beta + \delta, \alpha < \varepsilon + \lambda.$$

These conditions imply: $a < 0$, $b > 0$, $f > 0$, $a < d$, $b > c$, $e < f$, $c < f$.

In this Appendix, we adopt the same terminology used in Bomze (1983). By *an eigenvalue EV of a fixed point* we shall understand an eigenvalue of the linearization matrix around that fixed point. The term *EV in the direction of the vector V* means that V is an eigenvector corresponding to that *EV*. *IntS* is the set $\{x \in S : x_i > 0, i = 1, 2, 3\}$ in which all strategies are present in the population. An *edge* of S consists of all population states in which a given (fixed) strategy is not adopted; we shall denote $e_i - e_j$ the edge joining e_i with e_j, where $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$ represent the states in which only strategies *NSP, SPF* and *SPN* are played, respectively. Thus, e.g., $e_1 - e_2$ is the edge where only strategies *NSP* and *SPF* are present in the population.

Let us first observe that the states in which only one strategy is adopted by individuals, e_i, are always fixed points under replicator dynamics. Their stability properties are analyzed in the following proposition.[9] For simplicity, the propositions in Bomze (1983) will be indicated as B# (see, e.g., B4 is Proposition 4 of Bomze's paper).

[8]It is a well-known result that dynamics (3) does not change if an arbitrary constant is added to each column of A (see, e.g., Hofbauer & Sigmund, 1988, p. 126).

[9]All the eigenvalues of the fixed points on the edges of S are real (see Bomze, 1983).

Proposition 1. *The eigenvalue structure of the fixed points e_i, $i = 1, 2, 3$, is the following:*

1. *e_i has one eigenvalue with the sign of $a = \beta - \gamma - \alpha$ (<0 always) in the direction of $e_1 - e_2$ and one eigenvalue with the sign of $d = \varepsilon - \eta - \alpha$ in the direction of $e_1 - e_3$.*
2. *e_2 has one eigenvalue with the sign of $-b = \alpha - \beta - \delta$ (<0 always) in the direction of $e_1 - e_2$ and one eigenvalue with the sign of $e - b = \varepsilon - \beta - \delta$ in the direction of $e_2 - e_3$.*
3. *e_3 has one eigenvalue with the sign of $-f = \alpha - \varepsilon - \lambda$ (<0 always) in the direction of $e_1 - e_3$ and one eigenvalue with the sign of $c - f = \beta - \varepsilon - \lambda$ (<0 always) in the direction of $e_2 - e_3$.*

Proof. Apply B1. □

The following proposition concerns the fixed points on the edges of S.

Proposition 2.

1. *A unique fixed point always exists in the interior of $e_1 - e_2$. The eigenvalues of such a fixed point have the sign of $-a = -\beta + \gamma + \alpha$ (> 0 always) in the direction of $e_1 - e_2$ and of $(bd - ae)/b$ in the direction of the interior of S, where $(bd - ae)/b < 0$ if and only if (iff):*

$$\begin{aligned} bd - ae &= (\beta + \delta - \alpha)(\varepsilon - \eta - \alpha) - (\beta - \gamma - \alpha)(\varepsilon - \alpha) = \\ &= -\eta(\beta + \delta - \alpha) - (\gamma + \delta)(\alpha - \varepsilon) < 0. \end{aligned} \tag{5}$$

2. *A unique fixed point exists in the interior of $e_1 - e_3$ iff $d = \varepsilon - \eta - \alpha < 0$; no fixed point exists if such a condition does not hold. The eigenvalues of the fixed point in the interior of $e_1 - e_3$ (when existing) have the sign of $-d$ (>0 always) in the direction of $e_1 - e_3$ and of $(af - cd)/f$ in the direction of the interior of S, where $(af - cd)/f < 0$ iff:*

$$\begin{aligned} af - cd &= (\beta - \gamma - \alpha)(\varepsilon + \lambda - \alpha) - (\beta - \alpha)(\varepsilon - \eta - \alpha) = \\ &= -\gamma(\varepsilon + \lambda - \alpha) - (\eta + \delta)(\alpha - \beta) < 0. \end{aligned} \tag{6}$$

3. *A unique fixed point exists in the interior of $e_2 - e_3$ iff $(e - b)(f - c) = (\varepsilon - \beta - \delta)(\varepsilon + \lambda - \beta) < 0$ (i.e., iff $e - b = \varepsilon - \beta - \delta < 0$,* being $f > c$ always)*; no fixed point exists if such a condition does not hold. The eigenvalues of the unique fixed point in the interior of $e_2 - e_3$ have the sign of $(e - b)(f - c)/(e - b + c - f)$ (>0 always) in the direction of $e_2 - e_3$ and of $(bf - ce)/(e - b + c - f)$ in the direction of the interior of S, where $(bf - ce)/(e - b + c - f) < 0$ iff:*

$$\begin{aligned} bf - ce &= (\beta + \delta - \alpha)(\varepsilon + \lambda - \alpha) - (\beta - \alpha)(\varepsilon - \alpha) = \\ &= \lambda(\beta + \delta - \alpha) + \delta(\varepsilon - \alpha) > 0. \end{aligned} \tag{7}$$

Proof. Apply B2 and B5. □

The remaining proposition concerns the fixed points in the interior of S, where all strategies are adopted.

Proposition 3. *There is a unique fixed point in IntS iff (see formulae (5), (6) and (7)):*

$$bf - ce > 0 \; ae - bd > 0 \; cd - af > 0. \tag{8}$$

If such a condition does not hold, then there are no fixed points in IntS.

Proof. According to B6, a unique fixed point in the interior of S exists iff the expressions in (8) are all strictly positive or all strictly negative. A segment of non-isolated fixed points can exist only if the expressions in (8) are simultaneously equal to zero. So, to prove this proposition, we have to show that the expressions in (8) cannot be simultaneously equal to zero or strictly negative. Observe that $bf - ce \leq 0$ and $ae - bd \leq 0$ hold iff, respectively:

$$\beta + \delta - \alpha \leq \frac{\delta(\alpha - \varepsilon)}{\lambda} \tag{9}$$

$$\beta + \delta - \alpha \leq -\frac{(\gamma + \delta)(\alpha - \varepsilon)}{\eta} \tag{10}$$

since, by assumption, $\beta + \delta - \alpha > 0$, conditions (9) and (10) cannot be simultaneously satisfied. This proves the proposition. □

The above propositions allow us to give a complete classification of possible dynamic regimes under dynamics (2).

Case 1 ($d = \varepsilon - \alpha - \eta \geq 0$ and $e - b = \varepsilon - \beta - \delta \geq 0$). In this case, by Proposition 1, the fixed point e_3 (where all players adopt *SPN*) is locally attractive while e_1 and e_2 are saddle points. By Proposition 2, there is a unique fixed point in the edge $e_1 - e_2$, while there are no fixed points in the edges $e_1 - e_3$ and $e_2 - e_3$. Since $d = \varepsilon - \alpha - \eta \geq 0$ implies $\varepsilon - \alpha > 0$, the expression $bd - ae$ (see (5)) is strictly positive; therefore, the fixed point in the edge $e_1 - e_2$ has two strictly positive eigenvalues, that is, it is repulsive. Looking at all possible dynamic regimes shown in Bomze (1983), it is easy to check that these properties are compatible with a unique phase portrait in Bomze's classification—phase portrait (Bpp#, hereafter) number 42.[10] This dynamic regime is shown in Figure 2 of our article.

[10]When we say that a dynamic regime under equations (3) corresponds to a phase portrait Bpp# shown in Bomze's paper, we mean that the two portraits are "geometrically equivalent" (see Bomze, 1983, p. 205), that is, the former can be obtained from the latter by flow reversal, rotations and reflections of the simplex S.

Case 2 ($d = \varepsilon - \alpha - \eta \geq 0$ and $e - b = \varepsilon - \beta - \delta < 0$). In this case, by Proposition 1, the fixed points e_2 and e_3 (where all players adopt *SPF* and *SPN*, respectively) are locally attractive while e_1 is a saddle point. By Proposition 2, there is a unique fixed point in the edge $e_1 - e_2$ and in the edge $e_2 - e_3$ while there are no fixed points in the edge $e_1 - e_3$. As in Case 1, the expression $bd - ae$ (see (5)) is strictly positive; therefore, the fixed point in the edge $e_1 - e_2$ is repulsive. The unique phase portrait in Bomze's classification possessing these properties is Bpp37. This dynamic regime is shown in Figure 3 of our article.

Case 3 ($d = \varepsilon - \alpha - \eta < 0$ and $e - b = \varepsilon - \beta - \delta \geq 0$). In this case, by Proposition 1, the fixed points e_1 and e_3 are locally attractive while e_2 is repulsive; by Proposition 2, there is a unique fixed point in the edges $e_1 - e_2$ and $e_1 - e_3$, while no fixed point exists in $e_2 - e_3$. Only Bpp9, Bpp37 and Bpp38 are compatible with these features of dynamics. In Bpp9, there exists a fixed point in the interior of S (that is, condition (8) holds), which is, however, a repulsive point, and the fixed points in the edges $e_1 - e_2$ and $e_1 - e_3$ are saddles; the corresponding dynamic regime is illustrated in Figure 4 of our article. In Bpp37 and Bpp38, no fixed point exists in the interior of S (that is, condition (8) does not hold); in Bpp37, the fixed point in the edge $e_1 - e_2$ is repulsive, while that in $e_1 - e_3$ is a saddle; vice versa, in Bpp38, the stability properties of such points depend on the sign of the expressions in (8) (see Proposition 2).

Case 4 ($d = \varepsilon - \alpha - \eta < 0$ and $e - b = \varepsilon - \beta - \delta < 0$). In this case, by Proposition 1, all the fixed points e_1, e_2 and e_3 are locally attractive; by Proposition 2, there is a unique fixed point in the edges $e_1 - e_2$, $e_1 - e_3$ and $e_2 - e_3$. Only Bpp7 and Bpp35 are compatible with these properties. In Bpp7, there exists a fixed point in the interior of S (that is, condition (8) holds), which is, however, a repulsive point, and the fixed points in the edges $e_1 - e_2$, $e_1 - e_3$ and $e_2 - e_3$ are all saddles; the corresponding dynamics is illustrated in Figure 5 of our article. In Bpp35, no fixed point exists in the interior of S (that is, condition (8) does not hold); one fixed point in the edges is repulsive while the others are saddles (the stability properties of such points depend on the sign of the expressions in (8)—see Proposition 2).