Decision-Making Under Strong Uncertainty: Five Applications to Sunspot Theory and Neo-Schumpeterian Growth Theory

Paolo Giordani

Dipartimento di Economia Pubblica, Università di Roma "La Sapienza"

Supervisor: Prof. Guido Cozzi Co-Supervisor: Prof. Karl Shell

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Decision Making Under Uncertainty: A Macro User Guide

"A probability measure cannot adequately represent both the relative likelihoods of events and the amount, type and reliability of the information underlying those likelihoods" *(Epstein and Wang, 1994)*

0.1 Introduction

The classical distinction between 'risk' and 'uncertainty' traces back to Frank Knight (1921), and states that *risk* is associated with situations in which an objective probability distribution of all events relevant to decision making is known, while *uncertainty* characterizes choice settings in which that probability distribution is not available to the decision-maker (DM, henceforth). Two important remarks need to be made from the very beginning. First, this book is exclusively concerned with decision making under uncertainty. Hence, all the advancements in the theory of choice under risk, such as the (first) 'prospect theory' (Kahneman and Tversky (1979)) or the 'rank-dependent expected utility theory' (Quiggin (1982)), are here mostly neglected. The second remark is related to a terminological issue. In order to avoid confusion around the Knightian distinction, in literature the single term ‘uncertainty’ is frequently accompanied by an attribute. However, over the years several attributes have been used to qualify ‘uncertainty’ with respect to risk (in such a way that, paradoxically, confusion seems to have risen in some respects). Hence, according to the standard literature, throughout the book we will use the phrases ‘strong uncertainty’, ‘Knightian uncertainty’, ‘non-probabilistic uncertainty’ and, sometimes ‘strict uncertainty’ with exactly the same meaning: an objective probability distribution over all relevant events is not known by the DM.

The distinction between probabilistic and non-probabilistic contexts of choice has also been recognized and powerfully stressed by John M. Keynes. Particularly in the context of asset prices, Keynes (1936) has challenged the notion that the mathematical calculation of expected value has much to do with them. Since a sort of ‘radical’ (that is, non-probabilistic) uncertainty pervades most of our investment decisions, the so-called ‘animal spirits’ - defined as “a spontaneous urge to action rather than inaction” - play a key role in determining those decisions: “Most, probably, of our decisions to do something positive, the full consequence of which will be drawn out over many days to come, can only be taken as a result of animal spirits” (Keynes (1936), Ch.7, p161). Keynes has not ruled out the possibility that people make their decisions after a full, rigorous evaluation of all consequences; yet he has argued that, when this possibility does not exist - and it occurs quite frequently in the ordinary business of life - people are driven by ‘animal spirits’.

During the fifties a number of economists and statisticians have worked on decision theory under uncertainty, trying to develop a reasonable and/or a realistic decision rule in situations in which an objective probability distribution is not provided. In those years the literature on decision theory (see Milnor (1954) and Luce and Raiffa (1958)) has recognized four criteria under *complete ignorance*: the ‘maximin return criterion’ (Wald (1950)), the ‘minimax regret criterion’ (Savage

(1954, ch.9)), the 'optimism-pessimism index criterion' (Hurwicz (1951)) and the Laplace 'principle of insufficient reason' (as reconsidered by Milnor (1954) and Chernoff (1954)). Although they work in rather different ways and generally identify different 'optimal choices' (as we will see in the next Section), they have the common feaure of being *distribution-free* decision rules, in the sense of being able to find the optimal choice without requiring any probability distribution over the uncertain events (for this reason they are usually called 'decision criteria under complete ignorance').

During approximately the same years, a meaningful distinction between risk and uncertainty has been strongly challenged by an axiomatic approach to decision making under uncertainty, the so-called 'subjective expected utility (SEU) theory', proposed by Savage (1954) and deeply inspired by the previous works of de Finetti (1931) and Ramsey (1931). Broadly speaking, according to the SEU model it is not necessary to assume the existence of an objective probability distribution in order to make use of the expected utility criterion (axiomatized by Von Neumann and Morgenstern (1944)). Given a preference relation satisfying some 'reasonable' properties (or axioms), then a unique expected utility-based decision rule can be derived, in which choices are made *as if* individuals held probabilistic beliefs. As a matter of fact, also for reasons of tractability the SEU model has been representing the standard way of dealing with uncertainty in economic models over the last fifty years.

However, despite the supremacy of the SEU model in standard economic analysis, a number of criticisms have been raised over the years. One of the most famous and significant criticisms has been provided by Ellsberg (1961) via a rather simple mind experiment, generally known as the 'Ellsberg paradox'. As we will see in Section 0.3, this paradox illustrates a situation in which the common choice behavior of the (real) decision maker is incompatible with the one prescribed by the SEU model.

More recently the Knightian distinction has been receiving a growing attention from economists. A huge research effort has been devoted to elaborate a decision model, which could meaningfully distinguish between situations in which an objective probability distribution is given from situations in which it is not. A number of axiomatic approaches have been developed during the eighties and the first nineties, that have generalized the SEU model so as to embrace this distinction. Even if we suggest some alternative approaches, we are particularly interested in the Choquet expected utility (CEU) theory, axiomatized by Schmeidler (1989), and in the maximin expected utility (MEU) theory, axiomatized by Gilboa and Schmeidler (1989). Both of them are strongly related to the point raised by the Ellsberg paradox, even if they start from a cognitive rather than from a behavioral perspective.

Before concluding, another terminological remark is neccessary: in the context of CEU and MEU models, uncertainty is also (and perhaps more often) called *ambiguity*, which is the original term used by Ellesberg (1961). In what follows we will normally make use of the term 'uncertainty', in honor of David Schmeidler. However notice that, in this stream of literature, they have exactly the same meaning.

In this chapter we further develop the issues sketched out above, and provide the basic decision making *tools* that we will be applying in the next chapters. The rest of the chapter is organized as follows. In the next Section we shall analyze the four classic non-probabilistic decision criteria under complete ignorance. In Section 0.3 we shall introduce some axiomatic approaches to decision making under uncertainty (mainly SEU, CEU and MEU), present the Ellsberg paradox, and review some recent applications of these approaches to different fields of economic analysis. In the last Section we shall present the plan of the book and sum up the basic findings of the next chapters.

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0.2 Four Classic Distribution-Free Decision Rules

0.2.1 A Basic Formalization

In this Section we provide a basic introduction to the four standard decision rules under complete ignorance acknowledged in literature (see Milnor (1954) and Luce and Raiffa (1958)): the 'maximin return criterion' (MMC, Wald (1950)), the 'minimax regret criterion' (MMRC, Savage (1954, ch.9)), the 'optimism-pessimism index criterion' (OPIC, Hurwicz (1951)) and the Laplace 'principle of insufficient reason' (PIR, Milnor (1954)). All of them are to be included in the class of the non-probabilistic models of decision making under uncertainty, in that they allow decision makers (DMs) to determine their optimal choice (and, more generally, to express a weak preference order) in uncertain settings, without requiring them to know a probability distribution over all possible states of nature.

Assume, as usual, that the DM has a perfect knowledge of the entire set of the states, but that she is not able to evaluate the probability associated with the realization of each of them. Then a decision problem under complete ignorance can be easily represented through a 'decision table', such as the one in figure 1. A_i (for $i = 1, ..., m$) represents the generic act, S_j (for $j = 1, ..., n$) represents the generic state of nature, and c_{ij} is the consequence (pay-off) associated with act i and state j. We now present these four decision rules[1].

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States of nature

[1] Notice that we do not thoroughly deal with the basic properties and shortcomings of all these criteria. For a more complete treatment see, among the others, French (1986).

		S_1	S_2	...	S_n
	A_1	c_{11}	c_{12}	...	c_{1n}
Acts	A_2	c_{21}	c_{22}	...	c_{2n}
	...	...	...	...	...
	A_m	c_{m1}	c_{m2}	...	c_{mn}

Figure 1. *The generic decision table.*

The maximin return criterion. MMC works as follows (refer to table 1): for each act A_i, the decision maker identifies the minimum pay-off over the set of states of nature:

$$s_i = \min_{j\in[1,n]} \{c_{ij}\},$$

where s_i is usually called the 'security level' of act A_i. Then she chooses the act for which this security level is the highest:

$$\text{choose } A_k \text{ such that } s_k = \max_{i\in[1,m]} \{s_i\} = \max_{i\in[1,m]} \left\{ \min_{j\in[1,n]} \{c_{ij}\} \right\}. \qquad \text{(MMC)}$$

The minimax regret criterion. The decision process driven by MMRC can be split in three stages. In the first stage, the DM computes the regret associated with any given pair act/state by subtracting the consequence corresponding to that pair from the best consequence that is achievable in the same state, that is:

$$r_{ij} = \max_{l\in[1,m]} \{c_{lj}\} - c_{ij}$$

The result of this process is the 'matrix of regrets', which by construction has the same dimension as the starting decision table. In the second stage the DM associates with each act the maximum regret over all states of nature, that is:

$$\rho_i = \max_{j\in[1,n]} \{r_{ij}\}.$$

Finally she selects the act for which the maximum regret (associated with each act) is the smallest, that is:

choose A_k such that $\rho_k = \min_{i\in[1,m]} \{\rho_i\} = \min_{i\in[1,m]} \left\{ \max_{j\in[1,n]} \{r_{ij}\} \right\}.$ (MMRC)

The optimism-pessimism index criterion. In order to describe OPIC, we need to introduce first the 'maximax return criterion'. This is the optimistic counterpart of the maximin return in the sense that, for each act A_i, the decision maker identifies the maximum pay-off over the set of states of nature:

$$o_i = \max_{j\in[1,n]} \{c_{ij}\},$$

where o_i is now called the 'optimism level' of act A_i. Afterwards she chooses the act for which this optimism level is the highest:

choose A_k such that $o_k = \max_{i\in[1,m]} \{o_i\} = \max_{i\in[1,m]} \left\{ \max_{j\in[1,n]} \{c_{ij}\} \right\}.$

We only need the value o_i because, under OPIC, the return associated with each act A_i is computed according to a α-weighted average of the security level s_i and the optimism level o_i, for $0 \leq \alpha \leq 1$. Finally the act associated with the highest weighted sum is picked up, that is:

choose A_k such that $\alpha s_k + (1-\alpha)o_k = \max_{i\in[1,m]} \{\alpha s_i + (1-\alpha)o_i\}$ (OPIC)

The parameter α, roughly representing the DM's degree of pessimism, can of course vary across different individuals.

The Laplace principle of insufficient reason. This criterion states that, if the DM completely ignores the probability distribution associated with all states of nature, she will act as if these states were equiprobable. Hence, to evaluate each act A_i, the DM will compute the following expected utility:

$$e_i = \sum_{j=1}^{n} (\frac{1}{n}) c_{ij}. \qquad \text{(PIR)}$$

Afterwards she will select the act which maximizes her expected utility:

choose A_k such that $e_k = \max_{i\in[1,m]} \{e_i\} = \max_{i\in[1,m]} \left\{ \sum_{j=1}^{n} (\frac{1}{n}) c_{ij} \right\}$

As the reader can easily verify in the following example due to Milnor (1954), these criteria generally bring different choice results.

0.2.2 A Simple Example (Milnor (1954))

Assume that our uncertain scenario is made up of four alternative acts and four possible states of nature, such as the one represented in the following matrix.

States of nature

		S_1	S_2	S_3	S_4
	A_1	2	2	0	1
acts	A_2	1	1	1	1
	A_3	0	4	0	0
	A_4	1	3	0	0

Figure 2. *The decision table.*

The optimal choice associated with the maximin return criterion is A_2, since $s_2 = 1 > s_1 = s_3 = s_4 = 0$. In order to find the optimal choice under MMRC, we must build the matrix of regrets (for instance, the regret associated with the first column is computed as follows: $r_{11} = \max_{1 \leq l \leq 2} \{c_{lj}\} - c_{11} = 2 - 2 = 0$, $r_{21} = 2 - 1 = 1$, $r_{31} = 2 - 0 = 2$, $r_{41} = 2 - 1 = 1$).

States of nature

		S_1	S_2	S_3	S_4
	A_1	0	2	1	0
acts	A_2	1	3	0	0
	A_3	2	0	1	1
	A_4	1	1	1	1

Figure 3. *The matrix of regrets.*

It turns out that the best choice is A_4, since $\rho_4 = 1$ is lower than $\rho_1 = \rho_3 = 2$ and $\rho_2 = 3$.

The optimal choice under OPIC will obviously depend on the specific value of α. Given the following security and optimism levels for respectively acts 1, 2, 3 and 4: $(s_1, o_1) = (0, 2)$, $(s_2, o_2) = (1, 1)$, $(s_3, o_3) = (0, 4)$ and $(s_4, o_4) = (0, 3)$, the DM must choose among the following pay-offs: $2(1-\alpha)$, 1, $4(1-\alpha)$ and $3(1-\alpha)$ associated with respectively acts 1,2,3 and 4. Since $4(1-\alpha) > 3(1-\alpha) > 2(1-\alpha)\ \forall\alpha \in [0, 1)$, the agent will select A_3 if and only if: $4(1-\alpha) > 1 \rightarrow \alpha > \dfrac{3}{4}$. Then, for $\alpha \in [0, \dfrac{3}{4})$ and $\alpha = 1$, A_2 is strictly preferred; for $\alpha \in (\dfrac{3}{4}, 1)$, A_3 is strictly preferred and for $\alpha = \dfrac{3}{4}$, DM is indifferent between them

Finally PIR imposes A_1 as the optimal choice, since $e_1 = \frac{5}{4} > e_2 = e_3 = e_4 = 1$.

0.3 Axiomatic Approaches to Uncertainty

In this Section our main purpose is to present and compare different preference axiomatizations for decision making under uncertainty. We illustrate their main properties (and shortcomings), and describe, as simply as possible, the choice behavior prescribed by each of them. Hence, we do not thoroughly deal with the axiomatic foundations of the theories presented, even if we do carefully mention the key axioms underlining each of them. The next Subsection is devoted to the classic subjective expected utility (SEU) model (see, among the others, Savage (1954) and Anscombe and Aumann (1963)). Subsection 0.3.2 presents the Ellsberg paradox (Ellsberg (1961)). To a certain extent, it constitutes the starting point of the theoretical extensions dealt with in the rest of the Section. In Subsections 0.3.3 and 0.3.4 we analyze two axiomatizations generalizing the SEU model, respectively the Choquet expected utility (CEU) model (Schmeidler (1989)) and the maximin expected utility (MEU) model (Gilboa and Schmeidler (1989)). As we will see, both of them are based on a weakening of the 'independence axiom'. In Subsection 0.3.5 we hint at an alternative axiomatization, developed by Bewley (1986), which instead abandons the 'completeness axiom'. Finally, in the last Subsection we briefly review some recent applications of CEU and MEU to different fields of economic analysis.

0.3.1 The Subjective Expected Utility (SEU) Theory

The Knightian distinction between uncertainty and risk has been mostly neglected by the economics mainstream over the second half of the twentieth century. The growing prominence conquered by the expected utility principle in situations of uncertainty is largely due to the axiomatic foundation of the subjective probability assignments (see de Finetti (1931) and Savage (1954)). The SEU theory is a generalization of the Von Neumann-Morgenstern's (1944) theory of expected utility, in the sense that it extends its applicability to decision settings in which objective probabilities are not given. According to the SEU appraoch, the objective probability distribution of the relevant events may also be unknown to the decision maker because, in choosing among acts, she behaves *as if* she knew that law and selects the feasible act with the highest (subjective) expected utility. In this sense the SEU model seems to undermine any meaningful distinction between risk and uncertainty.

Now let us give a basic formalization of this decision rule. Given a set S and an algebra of subsets Σ in S, every element $s \in S$ is called state of nature or simply *state*, and every element E in Σ is called *event*. C denotes the set of *outcomes* - in the simplified SEU version of Anscombe-Aumann (1963) these outcomes are 'roulette lotteries' (objective). F is the set of *acts*, which are $\Sigma-$measurable finite-valued functions mapping states to outcomes, that is, $f : S \rightarrow C \quad \forall f \in F$ - in the Anscombe-Aumann's (1963) framework these are the 'horse lotteries' (subjective). The representation theorem in every SEU model proves that, given a preference relation $\succeq$ over acts satisfying certain properties (or axioms), there exists a unique subjective (additive) probability measure $p(\cdot) : \Sigma \to R$ and a Von Neumann-Morgenstern utility function $U(\cdot) : C \to R$ such that, for any two acts f and g in F:

$$f \succeq g \textit{ if and only if } \int_S U(f(\cdot))dp \geq \int_S U(g(\cdot))dp. \qquad \text{(SEU)}$$

In other words, the subjective expected utility form is able to *represent* the preference relation $\succeq$, if this relation satisfies some 'reasonable' axioms. Among them we recall the *weak order*, *continuity* and *independence*. These three axioms are common to the Von Neumann-Morgenstern's expected utility theory and to every axiomatization of SEU. Importantly, remember that they are necessary but not sufficient to derive the SEU form, as we have presented it in (SEU). The particular reinforcements, that are needed to univocally pin down that expression, depend on the particular model chosen[2]. The first axiom is a rationality requirement, and implies completeness and transitivity of the preference relation $\succeq$. The second is a technical condition necessary to guarantee the existence of a function representing $\succeq$. It requires that small changes in probabilities do not alter the order between two acts. More formally, given three acts f, g, h in F, if $f \succ g$ and $g \succ h$, then there are α and β in $(0, 1)$ such that: $\alpha f + (1 - \alpha)h \succ g$ and $g \succ \beta f + (1 - \beta)h$. Finally the independence axiom states that the preference order between two acts should not be affected if, for an event for which the two acts yield the same outcome, that common outcome is changed into another common outcome. Formally, given three acts $f, g, h \in F$ and a real number $\alpha \in [0, 1]$, then $f \succ g$ if and only if $\alpha f + (1 - \alpha)h \succ \alpha g + (1 - \alpha)h$. In the traditional Savage's framework this axiom is slightly different and usually called the *sure-thing principle*.

All of the three axioms stated above have come under discussion. However we are here mostly interested in the violations of the independence axiom (some brief considerations regarding the

[2] For example, in the Anscombe and Aumann's (1963) version of SEU, two additional axioms, namely 'state-independence of preferences' and 'non-degeneracy', are required to represent the preference relation via the subjective expected utility form.

weak order property will be given in Subsection 0.3.5). In 1961 Daniel Ellsberg elaborated a mind experiment, which is generally known as 'The Ellsberg Paradox'. This experiment effectively shows how incongruent real choices might be with respect to the choices prescribed by the SEU model, and justifies that divergence on the basis of the strong uncertainty characterizing the agent's choice setting. The emergence of the paradox is basically due to a violation of the independence axiom.

0.3.2 The Ellsberg Paradox

Two urns 1,2 are given, each of which contains ten balls. Urn 1 is known to contain five white balls and five black balls, while no information is given about the colors of the ten balls in urn 2. One ball is drawn at random from each of the two urns. The DM is asked to rank four possible bets, denoted: $1W$, $1B$, $2W$, $2B$, where $1W$ denotes the bet 'a white ball is drawn from urn 1', $1B$ the bet 'a black ball is drawn from urn 1' and so on. The DM will gain 100 euro if she wins the bet and 0 otherwise.

The paradox arises because most people show the following preference order: $1W \sim 1B \succ 2W \sim 2B$. That is, they are indifferent as the color to bet on in both urns, but strictly prefer to bet on the 'known' urn rather than in the 'unknown' urn. In this sense, they show a sort of 'preference for objective probabilities'. This choice behavior cannot be explained in the SEU framework, since there is no subjective (additive) probability distribution that supports these preferences.

To accomodate Ellsberg-type behavior, it is indeed necessary to abandon the standard SEU framework and to enter into a more general representation of choice under uncertainty. This is exactly what we will do in the rest of the Section. For further emprical evidence against the SEU model see, among the others, Camerer and Weber (1992).

Interestingly, this paradox reminds us of the distinction between *probability* and *weight of evi-*

dence elaborated by Keynes (1921) in his "Treatise of Probability" . While the 'probability' represents the balance of evidence in favor of a particular proposition, the 'weight of evidence' stands for the quantity of information supporting that balance. In our case the prevailing choice of urn 1 is not due to a 'better' probability distribution but to a superior weight of evidence that supports it. As a matter of fact, the difference between clear and vague probabilities has been rejected by the subjectivist school, and the paradox finally arises because "the probability attached to an uncertain event does not reflect the heuristic amount of information that led to the assignment of that probability" (Schmeidler (1989)).

0.3.3 The Choquet Expected Utility (CEU) Theory

The Choquet expected utility (CEU) model has been axiomatized by Schmeidler (1989) in the Anscombe-Aumann's (1963) framework. It is a generalization of the subjective expected utility theory, which can accommodate the choice behavior of Ellsberg-type situations. The extension consists of allowing for *non-additive probability distributions*. As is well known, in the SEU model subjective beliefs are represented via an additive probability function $p(\cdot)$, whose three basic properties are: (i) $p(\varnothing) = 0$; (ii) $p(S) = 1$ and, given two generic events E, F in Σ (iii) $p(E \cup F) + p(E \cap F) = P(E) + p(F)$. A non-additive probability function $\pi(\cdot)$, also called a *capacity*, is a generalization of the additive one, in the sense that it satisfies the first two properties and replaces the last one with the following weaker property: given two events E, F in Σ, $E \subseteq F$ implies $\pi(E) \leq \pi(F)$. As we will see, a non-additive probability measure is able to reflect the Keynesian 'weight of evidence', that is, the amount of information used to evaluate the probability of an event.

In decision theory the idea of non-additive probability distributions was not new at the time it was introduced by Schmeidler (1989). In a context of decision making under pure risk Kahneman and Tversky (1979), in the first version of their *Prospect Theory*, had introduced a probability weighting function, which transformed the individual probabilities directly into decision weights. However, their formalization presented serious analytical difficulties, since the decision rule violated continuity and monotonicity (i.e., the first order stochastic dominance). One of the great merits of Schmeidler has been to overcome this problem by introducing the *Choquet integral* (Choquet (1955)) in order to compute the expected utility. The Choquet integral of a measurable function $f : S \to R$ with respect to a non-additive probability distribution $\pi(\cdot)$ can be defined as follows[3]:

$$CI_\pi(f) = \int_S f \; d\pi = \int_{-\infty}^{0} \left[\pi(f \geq \phi) - 1\right] d\phi + \int_{0}^{\infty} \pi(f \geq \phi) d\phi \qquad \text{(CI)}$$

The CEU model is an axiomatic theory. It states some properties to be satisfied by the preference ralation $\succeq$, and derives the (unique) expected utility form which represents it. The key difference with respect to the Anscombe-Aumann's (1963) model is the presence of the *co-monotonic independence axiom*, which is a weakening of the standard independence axiom. Two acts f and g are said to be co-monotonic if, for every two states s and t, it never happens that $f(s) \succ f(t)$ and $g(s) \prec g(t)$. This new axiom states that, given three *co-monotonic* acts $f, g, h \in F$ and a real number $\alpha \in \,]0, 1[$, then $f \succ g$ implies $\alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$. By substituting for this weaker version of the independence axiom into the Anscombe-Aumann's framework, Schmeidler (1989) proves[4] that there exists a unique non-additive probability measure $\pi(\cdot)$, and a Von Neumann-Morgenstern utility function $U(\cdot)$, such that the resulting Choquet expectation represents the preference relation $\succeq$. Formally, for any two acts f and g in F :

$$f \succeq g \textit{ if and only if } \int_S U(f(\cdot))d\pi \geq \int_S U(g(\cdot))d\pi \qquad \text{(CEU)}$$

[3] We will provide a computational example of Choquet expectation, in the simple case in which the function f takes finitely many values, later in this Subsection. Notice that in that case the Choquet expectation can be computed as follows:

$$CE_\pi(f) = \int_s f d\pi = f(s_1) + \sum_{i=2}^{N} \left[f(s_i) - f(s_{i-1})\right] \cdot \pi\left(\{s_i, ..., s_i\}\right),$$

where the outcomes are arranged in increasing order: $f(s_1) \leq ... \leq f(s_N)$.

[4] Indeed he also replaces the 'strict monotonicity axiom' of Anscombe and Aumann with the 'monotonicity axiom'

where the integral is now a Choquet integral. According to Schmeidler (1989), a CEU-maximizer is said to be *uncertainty averse* if her subjective beliefs are represented by a *convex* capacity. Convexity means that, for any two events E, F in Σ, it holds $\pi(E \cup F) + \pi(E \cap F) \geq \pi(E) + \pi(F)$. This property states that the (non-additive) probability of a set cannot be smaller than the sum of the (non-additive) probabilities of the cells of the partition of that set. In particular, the uncertainty (μ) of the belief about an event E can be measured by the expression $\mu(\pi(E)) = 1 - \pi(E) - \pi(E^c)$. Importantly, notice that this is a measure associated with the probability of a single event, not with the whole subjective prior. In other words it is not generally possible to compare two different priors in order to state which one is more uncertain. This definition of uncertainty aversion (together with the one that will be given in the subsequent Section) has recently been questioned, and some other definitions have been proposed[5].

Example 1 *The following example, drawn from Mukerji (1998), illustrates how to compute the Choquet expectation (CE) in a rather simple case. Consider a state space* Ω *partitioned into two events,* E *and* E^c*. Also assume that* E *consists of two possible events,* E_1 *and* E_2*, and that* E_1 *is in turn composed of* E_{11} *and* E_{12}*. The folllowing non-additive probability distribution is given:* $\pi(E_{11}) = 0.1$*;* $\pi(E_{12}) = 0.2$*;* $\pi(E_1) = 0.5$*;* $\pi(E_2) = 0.2$*;* $\pi(E) = 0.8$*;* $\pi(E^c) = 0.2$*;* $\pi(E_{11} \cup X) = \pi(E_{11}) + \pi(X)$*;* $\pi(E_{12} \cup X) = \pi(E_{12}) + \pi(X)$ *where* $X = \{E_2, E^c, E_2 \cup E^c\}$*;* $\pi(E_1 \cup E^c) = \pi(E_1) + \pi(E^c)$*;* $\pi(E_2 \cup E^c) = \pi(E_2) + \pi(E^c)$*;* $\pi(\Omega) = 1$*. Finally the pay-offs associated with the act* f *are as follows:* $f(E_{11}) = 10$*;* $f(E_{12}) = 20$*;* $f(E_2) = 5$*;* $f(E^c) = 0$*. Now order all pay-offs in decraesing order and compute the expected value of the act* f *with respect to* π *as follows:* $CE_\pi(f) = f(E_{12})\pi(E_{12}) + f(E_{11})\left[\pi(E_1) - \pi(E_{12})\right] + f(E_2)\left[\pi(E) - \pi(E_1)\right] + f(E^c)\left[\pi(\Omega) - \pi(E)\right] = 8.5$*. It is*

[5] See, among the others, Epstein (1999), Ghirardato-Marinacci (2002), and Ghirardato et al. (2004). For a brief overview of the issue see also Ghirardato (2003).

even more intuitive to compute the Choquet expectation in the following alternative way:

$CE_\pi(f) = \pi(E_{11})f(E_{11}) + \pi(E_{12})f(E_{12}) + \pi(E_2)f(E_2) + \pi(E^c)f(E^c) + [\pi(E_1) - \{\pi(E_{11}) + \pi(E_{12})\}] \min\{f(E_{11}), f(E_{12}$

$[\pi(E) - \{\pi(E_1) + \pi(E_2)\}] \times$

$\min\{f(E_{11}), f(E_{12}), f(E_2)\} = 8.5.$

The first four addends in the expression above closely resemble the standard expectation operator. The crucial difference lies in the fact that, since the capacity π is convex, the two 'residual' probabilities ($[\pi(E_1) - \{\pi(E_{11}) + \pi(E_{12})\}]$ and $[\pi(E) - \{\pi(E_1) + \pi(E_2)\}]$) in each subset of events must be attached to the worst pay-off inside that subset.

The choice behavior in the Ellsberg's experiment can be explained if the agent's subjective probability is a convex capacity. To verify it, consider as an example that a DM, who has shown a preference for betting on the 'known' urn 1, presents the following convex probability distribution over the 'unknown' urn 2: $\pi(2W) = \pi(2B) = 0.47$, and the following additive probability distribution over the 'known' urn 1: $\pi(1W) = \pi(1B) = 0.5$ The convexity of the first distribution is easily verified: $\pi(2W) + \pi(2B) = 0.94 < \pi(W \cup B) = 1$, where $2W$ and $2B$ are two disjoint events. If we compute the Choquet expectation over the two urns, we get $CE_\pi(1) = 0.5 \cdot 100 + 0.5 \cdot 0 > CE_\pi(2) = 0.47 \cdot 100 + (1 - 0.47) \cdot 0$. Given those subjective beliefs, the strict preference for urn 1 is now perfectly rational.

0.3.4 The Maximin Expected Utility (MEU) Theory

MEU theory has been axiomatized by Gilboa and Schmeidler (1989) in the Anscombe-Aumann's (1963) version of the SEU model. In representing subjective beliefs, it suggests to replace the standard single (additive) prior $p(\cdot)$ with a closed and convex set Δ of (additive) priors. For this reason MEU model belongs to the class of *multi-prior* models. The choice among alternative acts is made according to the maximin strategy: for each act the DM first computes the minimal expected utility over the set Δ of priors, and finally singles out the act associated with the highest computed value. According to this model, the agent is said to be *uncertainty averse* if the set of priors is not a singleton. The decision problem can then be visualized as a 'two-player zero-sum game' characterized by:

- the *minimizing* behavior of a 'malevolent Nature', which selects the prior belief associated with the 'worst possible scenario' inside a pre-specified set of priors and
- the EU-*maximizing* behavior of the agent, whose optimal choice must take into account the worst-case strategy implemented by Nature.

The two key axioms that lead to the new representation theorem are:

1. the 'uncertainty aversion' axiom, which requires that the decision maker weakly prefer to randomize her choice among mutually indifferent acts: i.e for any two acts f, g in F, $f \sim g \rightarrow \alpha f + (1-\alpha)g \succeq f \ \forall \alpha \in \,]0,1[\,$.
2. The 'c-independence' axiom, a weakening of the classical independence axiom, which requires that an act f be preferred to an act g if and only if any mixture of f with a *constant* act h is preferred to the same mixture between g and h.

The representation theorem states that, given a preference relation $\succeq$ satisfying certain axioms (among which we have recalled the two key axioms above), then there exists a unique closed and convex set Δ of finitely additive probability measures $p(\cdot)$ on Σ, and a Von Neumann-Morgenstern utility function $U(\cdot) : C \rightarrow R$ such that the maximin subjective expected utility form represents $\succeq$, that is, such that for any two acts f and g:

$$f \succeq g \textit{ if and only if } \min_{p\in\Delta} \int_S U(f(\cdot))dp \geq \min_{p\in\Delta} \int_S U(g(\cdot))dp \qquad \text{(MEU)}$$

Think again of the Ellsberg Paradox and assume that the DM shows the SEU-incompatible preference order: $1W \sim 1B \succ 2W \sim 2B$. To verify the compatibility between this preference order and the MEU model, assume that subjective beliefs in the known urn 1 are represented by the single additive prior $[0.5; 0.5]$, where the first number stands for the probability of drawing a white ball, while in the unknown urn 2 they are represented by a range of priors, such as the following one: $[(0.47, 0.53)\,; (0.47, 0.53)]$, where the first interval is the probability interval associated with the event 'drawing a white ball'. When deciding a bet on urn 2, the DM always picks up the worst prior in computing expected utility, and then she compares the two following expected utility values: $MEU(1) = SEU(1) = 0.5 \cdot 100 + 0.5 \cdot 0 > MEU(2) = 0.47 \cdot 100 + 0.53 \cdot 0$. As a result, the DM always prefers to bet on the known urn 1.

There is a close relationship between MEU and CEU, if the non-additive probability used by the CEU-maximizer is convex. In order to analyze the basic similarities and differences between them, we need to introduce the concept of core. Given a convex capacity $\pi(\cdot)$, and given the set Λ of all possible finitely additive priors on the state space S, the *core* of $\pi(\cdot)$, denoted by $\Pi(\pi)$, can be defined as the set of finitely additive priors that majorize $\pi(\cdot)$ point-wise, that is: $\Pi(\pi) = \{p_i \in \Lambda : p_i(E) \geq \pi(E) \quad \forall\ E \in \Sigma\}$. As we have seen in the previous Subsection, the CEU-maximizer computes expected utility by means of the Choquet integral as defined in (CI). By definition, this value corresponds to the minimum of all possible expected values computed according to all additive priors inside the core $\Pi(\pi)$:

$$CE_\pi(f) = \min_{p\in\Pi(\pi)} \int_S U(f(\cdot))dp.$$

Hence CEU under a convex capacity and MEU bring exactly the same results if the range of priors Δ equals the core $\Pi(\pi)$ of the non-additive prior.

$\lambda_t \lambda_{t+1}$

0.3.5 Multiple Priors with Unanimity Ranking

CEU and MEU axiomatizations are both based on a weakening of the independence axiom in the SEU version of Anscombe and Aumann (1963)[6]. However independence is not the unique axiom to have been questioned over the years. In particular, in this Subsection we briefly introduce the model developed by Bewley (1986, 2003). He has axiomatized his decision rule by dropping the axiom of complete preferences inside the Anscombe and Aumann's (1963) framework. Assume, as we did under MEU theory, that the DM is endowed with a closed and convex set Δ of additive priors p. As we have just seen, in evaluating each act MEU suggests to replace this set with the minimizing prior. Bewley alternatively proposes to retain the whole set of priors and to compute, for each act, the expected utility with respect to all of them. Then, he states, an act f is preferred to an act g if, *for every prior* p inside Δ, the expected utility associated with f is above the one associated with g. More formally:

$$f \succ g \textit{ if and only if } \int_s U(f(\cdot))dp > \int_s U(g(\cdot))dp \;\; \forall p \in \Delta. \tag{MPUR}$$

This criterion reminds us of the 'strict domination' principle since, in order to be preferred, the act f must strictly dominate (in terms of expected utility) the act g for all possible p. Because of that, it only induces a partial ordering: the reader can easily imagine a situation in which f is preferred to g with respect to a certain subset of priors $\Gamma \subset \Delta$, while the converse is true with respect to its complement Γ^c. The violation of the *completeness axiom* has been considered as a weak point of this model by some decision theorists: even for purely practical reasons, it is problematical to handle situations in which the economic agent is simply not able to decide. Bewley answered to this

[6] The CEU model has been axiomatized in the original Savage's framework by Gilboa (1987).

criticism by elaborating the idea of the existence of a *status quo* act f_0, which is always preferred 'by default' unless another act dominates it in the sense clarified above. Unfortunately Bewley has not still developed a theory of how this status quo is generated.

0.3.6 Economic Applications of CEU and MEU

CEU and MEU models, as axiomatized by Schmeidler (1989) and Gilboa-Schmeidler (1989), have been largely applied to a wide range of research fields in economics. In this Section we will briefly review some applications respectively related to contract theory, finance and game theory[7].

In contract theory Mukerji (1998) has shown that the uncertainty aversion of the agent, as formalized by the CEU model, can (i) be one of the reasons for the contracts' incompleteness and (ii) provide an explanation for the puzzle related to the firm's choice between vertical integration and incentive contracts. With respect to the second point, notice in fact that ambiguity makes vertical integration much more widespread than predicted by standard theory, and this accords well with recent empirical findings. One common result to this stream of literature (see also Mukerji (2002)) is that uncertainty aversion leads to optimal contracts with lower powered incentives (than those determined by the standard theory). Also this point is supported by empirical work.

The range of applications in financial economics is particularly rich. Dow and Werlang (1992) have applied the CEU decision rule to the portfolio choice and, in a model with one safe and one risky asset, have proven that there exists a non-degenerate price *interval* at which a CEU agent will strictly prefer to take a zero position in the risky asset (that is, she neither buys nor sells short the asset). This proposition seems consistent with the empirical findings, and generalizes the standard result (based on SEU agents), in which that interval collapses to a point. Epstein and Wang (1994) extend

[7] For a more complete survey of economic applications see Mukerji and Tallon (2003).

that proposition by analyzing the effects of knightian uncertainty on the equilibrium asset prices in a general equilibrium framework. In an intertemporal pure-exchange economy a là Lucas (1978), they assume an uncertainty averse representative agent (via the MEU model). Their contribution is twofold. On the one hand, they provide the first extension of the MEU model to a dynamic setting. On the other hand, they find that the indeterminacy of the general equilibrium solution cannot be excluded. In other words, the existence of the knightian uncertainty in the fundamentals of the economy may lead to a continuous range of equilibria for security prices. This result could provide an explanation for the 'excess volatility puzzle', that is, for the fact that empirical analysis has been showing that real asset prices are more volatile than predicted by canonical asset pricing models. As stated by Epstein and Wang, "the non-uniqueness of prices and its 'origin' in the multiplicity of underlying priors accord well with Keynes's intuition"; that is to say, the indeterminacy calls attention to the role played by 'animal spirits' in both determining the particular equilibrium that the economy eventually reaches and justifying the sizeable volatility of asset prices.

The Schmeidler's approach has also stimulated a variety of applications in game theory. Much theoretical effort has been devoted to the definition of the new solution concepts, such as the notion of strategic equilibrium, under ambiguity in complete information normal form games (see among the others Dow and Werlang (1994), Lo (1996) and Marinacci (2000)). The consideration of ambiguity on the players' beliefs has also brought new insights in different applied contexts, such as auction theory (Lo (1998)) and voting behavior analysis (Ghirardato and Katz (2002)). Finally it is worth mentioning the paper by Eichberger and Kelsey (2002), which analyzes the effect of ambiguity on a public good contribution game. Yet much work remains to be done in game theory under Knightian uncertainty.

An approach based upon a similar intuition as the one which inspired the Schmeilder's work is the one developed by Hansen and Sargent in a number of recent contributions[8]. The starting point of these authors is rather different though, being the 'rational expectations (RE) hypothesis'. As is well known, one of the building blocks of the RE approach is the idea that the agent and the model builder share the same knowledge about the model. Now, since the latter normally recognizes that her model is just an approximation of the 'true model', then the agent must correspondingly share the same doubts about its correct specification. Hansen and Sargent claim that the agent copes with this 'model uncertainty' (or misspecification) by using a 'robust decision rule' which, although applied in a different framework, closely resembles the Schmeidler's MEU model: the decision maker 'perceives' a number of alternative models (as expressed via multiple prior beliefs), and maximixes her expected pay-off under the worst possible model. The decisions, originating from the application of the maximin strategy, turn out to be robust against possible misspecifications.

0.4 Plan of the Book

In the chapters that follow we shall provide five applications of the decision rules that we have been introducing in this chapter. *Part I* is concerned with applications to sunspot theory. Sunspot theory has formalized the idea that, in some circustances, economic fundamentals cannot be sufficient to pin down univocally the equilibrium allocation, and that the Keynesian 'animal spirits' can eventually matter. In this framework, strong uncertainty seems to us a promising way to qualify purely 'extrinsic uncertainty' (that is, uncertainty not related to fundamentals), and to represent the possibility of an agent's 'fuzzy perception' of the sunspot activity.

In *chapter 1* we consider a two-period, sunspot, pure-exchange economy a là Cass and Shell (1983), and analyze the case in which agents do not have a probabilistic knowledge of the 'sunspot

[8] Among the others, we recall Hansen, Sargent and Tallarini (1999), and Hansen and Sargent (2001).

activity'. Two generations, each of which is made up of identical agents, populate this economy. The participation in the Arrow securities market is restricted and the generation, which is allowed to trade in assets, can alternatively confront the uncertainty via two standard distribution-free decision rules under 'complete ignorance' (Luce and Raiffa (1958)): the 'minimax regret criterion' (Savage (1954), ch.9) and the 'maxmin return criterion' (Wald (1950)). When the former is used, then sunspots can matter. In particular we prove that, if the economy admits two Walrasian equilibria, then a unique sunspot equilibrium always exists. We pin down this equilibrium, determine the prices of the Arrow securities and show that, at these prices, no trade in securities takes place. In the same framework we prove that, with agents using the maxmin return criterion, sunspots do not matter.

In *chapter 2* we provide an application of the Gilboa-Schmeidler's MEU decision rule to the standard literature on 'bank runs', as started with the seminal Diamond and Dybvig (1983). In particular, we consider the banking model elaborated by Peck and Shell (2003), in which a broad class of feasible contractual arrangements is allowed and which admits a run equilibrium, and stress the assumption that depositors are uncertain of their position in the queue when expecting a run. Given MEU maximizing depositors, we prove that there exists a positive measure set of subjective prior beliefs, obtained from the minimization over the set of admissible priors, for which the bank run equilibrium disappears. The implication is that 'suspension schemes' are valuable since, in addition to improving risk-sharing among agents (Wallace (1990)), they may undermine panic-driven bank runs.

Part 2 is concerned with the neo-Schumpeterian growth theory (SGT). In SGT, the Schumpeter's view of economic development, as spurred by incessant R&D races aimed at gaining monopoly profits, is incorporated into an Arrow-Debreu dynamic general equilibrium framework with 'measurable uncertainty' (risk). The assumption of a perfectly assessable investment horizon - that is, the idea that transparent and well-organized financial markets allow savers to finance R&D activity in the light of an expected discounted value of future returns 'revealed' by an efficient stock market - is standard along Schumpeterian growth models. However, the innovation process is probably one of the most intrinsically uncertain economic activities. The mere observation of reality suggests that investors are not generally able to evaluate exactly the expected returns from R&D activity. In the next three chapters we will be concerned with R&D-driven growth models, in which some sort of (strong) uncertainty on the agents' beliefs about these returns is diversely introduced and formalized. Notice that this idea is quite close to the Schumpeter's original view of the innovative process as the breaking of a stationary equilibrium brought about by 'resourceful' entrepreneurs and, hence, as a process characterized by some intrinsically unpredictable aspect.

In *chapter 3* we claim that investment decisions on R&D activity are actually taken under conditions of strict uncertainty on their possible returns. As is well known, in the standard neo-Schumpeterian growth theory the arrival of innovation in the economy is governed by a Poisson process. The parameter of this process λ, representing the flow probability of innovation, is assumed to be perfectly known by investors. In this paper we explore the theoretical implications of the - rather realistic - possibility that investment decisions on R&D activity be taken under conditions of strong uncertainty on their possible returns. In the framework developed by Aghion and Howitt (1992) we then remove the hypothesis of a perfectly known λ, and assume that neither its exact value nor a prior distribution over its potential values is known by investors when deciding upon R&D investments. The investment decision process under complete ignorance is alternatively modeled via the four distribution-free choice criteria surveyed in Section 0.2. The equilibrium R&D efforts in steady-state are then determined under all these decision rules, and finally compared with each other and with the standard Aghion-Howitt solution. Comparative statics and welfare analysis are also carried out, and provide results in accordance with the original model. This paper represents an attempt to extend a standard Schumpeterian framework in order to account for the lack of information characterizing the returns on R&D investments, and proves the robustness of this framework to the investors' strong uncertainty.

In *Chapter 4* we provide a re-foundation of the symmetric growth equilibrium characterizing the research sector of vertical R&D-driven growth models (such as Grossman and Helpman (1991) and Howitt (1999)). This result does not rely on the usual assumption of a symmetric expectation on the future per-sector R&D expenditure. On the contrary, future per-sector distribution of R&D efforts is assumed to be strictly uncertain to the decision maker. By using the Gilboa-Schmeidler's MEU decision rule, we prove that the symmetric structure of R&D investment is the unique equilibrium compatible with uncertainty averse agents adopting a maximin strategy.

Finally in *chapter 5*, we develop a quality-ladder growth model with asymmetric fundamentals. In this model the steady-state analysis reveals that an asymmetric composition of expected R&D efforts is actually required in order to make the engaging in R&D in each sector equally profitable. However, if returns in R&D are equalized, agents turn out to be indifferent as to where targeting research and, hence, the problem of the allocation of R&D investments across sectors is indeterminate. As in chapter 4, to solve this problem we assume that the agents' beliefs on the future composition of R&D efforts are characterized by strong uncertainty and formalize their attitude towards uncertainty once again via the MEU model. With this assumption we provide a refoundation of the rational expectations equilibrium, in which actual and expected R&D efforts are equal among each others and are such that returns are equalized across sectors.

Part I

Strong Uncertainty in Sunspot Theory

“Most, probably, of our decisions to do something positive, the full consequence of which will be drawn out over many days to come, can only be taken as a result of animal spirits” *(John Maynard Keynes, 1936).*

Chapter 1

Do Sunspots Matter Under Complete Ignorance?

1.1 Introduction

In a seminal paper Cass and Shell (1983, JPE) prove that, in a simple general equilibrium model with overlapping generations, 'extrinsic uncertainty' - that is, uncertainty not affecting fundamentals - may play a role in determining the equilibrium allocation. In their model 'sunspots matter' because the overlapping-generations structure of the model brings about restricted participation in the Arrow securities market[1].

We develop a two-period pure-exchange general equilibrium model much in the spirit of Cass and Shell (1983). Two generations, each of which is made up of identical agents, populate the economy

[1] In the same work the authors also prove that, even though participation is not restricted, sunspots can matter if economic agents have heterogeneous beliefs on the 'sunspot activity'.

and the participation in the Arrow securities market is restricted to the one born in the first period. 'Strong uncertainty' seems a promising way to qualify purely 'extrinsic uncertainty' and to represent the possibility of an agent's 'fuzzy perception' of the sunspot activity[2]. In our model the agents trading in assets are not able to evaluate the probability of the realization of the different states of nature generated by 'extrinsic uncertainty'. In this choice scenario of *complete ignorance* (Luce and Raiffa (1958)), these agents can alternatively select their optimal consumption bundle via two standard non-probabilistic decision rules: the 'minimax regret criterion' (MMRC henceforth, see Savage (1954), ch.9) and the 'maxmin return criterion' (MMC, see Wald (1950))[3].

We show that, in an economy populated by decision makers who care about minimizing their maximum regret, sunspots matter. In particular, we prove that, in an economy admitting two Walrasian equilibria, a unique sunspot equilibrium always exists. We determine the equilibrium prices of the Arrow securities and show that, at these prices, no trade in securities will take place. In the same framework we prove that, with the agents confronting extrinsic uncertainty via the 'maxmin return criterion', sunspots do not matter.

The rest of the chapter is organized as follows. In the next Section we describe our simple pure-exchange economy. In Section 4 we prove our results.

1.2 The Model

We consider a simple pure-exchange economy lasting two periods, $\tau = 0, 1$ and characterized by l commodities, and two states of nature, $s = \alpha, \beta$. The uncertainty generated by the existence of

[2] A first attempt to introduce strong uncertainty in the evaluation of the 'sunspot activity' has been developed by Tallon (1998): in his model the agents are assumed to be Choquet-expected-utility maximizers (see Schmeidler (1989)).

[3] A treatment of the MMC and the MMRC in a general equilibrium framework has been provided by Pazner-Schmeidler (1975).

these two states is 'extrinsic', in the sense that it does not affect any fundamentals (preferences and endowments). There are two distinct generations of identical agents, G_0 born in period 0 and living to the end of time, and G_1 born in period 1 and also living to the end of time. The agents of both generations evaluate their consumption bundles via smooth, strictly increasing and strictly concave utility functions $U_h^{\alpha}(\cdot) \equiv U_h^{\beta}(\cdot) \equiv U_h(\cdot)$ for h in G_0,G_1. Endowments are represented by the vector $\omega_h(s) \equiv \omega_h$, while consumption bundles by $x_h(s)$, for h in G_0, G_1, $s = \alpha, \beta$. We denote the prices of the l commodities as the vector $p_c(s)$.

The timing of the model is the same as in Cass and Shell (1983). After their birth in period 0, the agents of generation G_0 are allowed to trade in Arrow securities, which are contingent on the realization of the extrinsic random variable. The amount of the s-contingent security bought - sold, if negative - by agent h in G_0 is $b_h(s)$ and its price is $p_b(s)$. At the end of period 0, before the birth of generation G_1, sunspot activity is observed (that is, people realize which state of nature has actually occurred). When both generations are alive in period 1, they trade in spot commodities and, finally, consume their bundles. The main feature of this scheme is that participation in the securities market is restricted to the agents in G_0. As is well known, with completely extrinsic uncertainty, if an equilibrium exists in which $x_h(\alpha) \neq x_h(\beta)$ for some h, then sunspots matter.

1.3 Do Sunspots Matter under MMRC and MMC?

Suppose that, for given fundamentals, the economy described above admits two distinct Walrasian equilibria, and that the 'extrinsic uncertainty' the agents in G_0 perceive corresponds to these two equilibria. We then index them as equilibrium 'α' with quantities and prices respectively given by $x_h^*(\alpha), p_c^*(\alpha)$, and equilibrium '$\beta$' with quantities and prices respectively given by $x_h^*(\beta), p_c^*(\beta)$, for h in G_0,G_1.

As we have argued above, agents in G_0 do not know the probability distribution over the two states of nature α,β, and their choice under 'complete ignorance' is alternatively driven by the MMRC and the MMC. Let us verify whether any other - sunspot-driven - equilibrium exists in this economy. Since G_1-type agents make their consumption choices after 'extrinsic uncertainty' has been resolved, they simply maximize their utility function under the usual budget constraints. On the contrary, G_0- agents can trade in Arrow securities (only) among each others and, then, must decide whether and, possibly, which amount $b_h(s)$ of assets to buy/sell in period 0, before sunspot activity is revealed.

Let us define the state-contingent pay-offs among which agent h in G_0 can choose. If agent h selects her optimal amount $b_h(\cdot)$ of Arrow security for a given price vector $p_b(\cdot)$, the pay-off she obtains can be summarized by the following indirect utility functions:

$v_h^\alpha = v_h\left[p_c(\alpha), p_c(\alpha)\omega_h + b_h(\alpha)\right]$ if α occurs and:

$v_h^\beta = v_h\left[p_c(\beta), p_c(\beta)\omega_h + b_h(\beta)\right]$ if β occurs.

where:

$$-\omega_h p_c(\alpha) \leq b_h(\alpha) = -b_h(\beta)\frac{p_b(\beta)}{p_b(\alpha)} \leq p_c(\beta)\omega_h\frac{p_b(\beta)}{p_b(\alpha)}.$$

In particular, if this agent decides to employ all her income in buying a positive amount of α-contingent security at the price $p_b(\alpha)$, her return is[4]:

$$v_h^{\alpha(\max)} = v_h\left[p_c(\alpha), \omega_h\left(p_c(\alpha) + \frac{p_b(\beta)}{p_b(\alpha)}p_c(\beta)\right)\right]$$

if state α occurs and

$v_h[p_c(\beta), 0]$

[4] These functions are determined by solving, for $s, t = \alpha, \beta$ and $s \neq t$ the following maximum problem:
$\max_{x_h} U[x_h(s)]$
s.t. $p_c(s)x_h(s) = p_c(s)\omega_h + b_h(s)$,
s.t. $b_h(s) = p_c(t)\omega_h\frac{p_b(t)}{p_b(s)}$.

if state β occurs. Analogously, if she decides to employ all her income in buying a positive amount of β-contingent security at the price $p_b(\beta)$, her return is:

$$v_h^{\beta(\max)} = v_h\left[p_c(\beta), \omega_h\left(p_c(\beta) + \frac{p_b(\alpha)}{p_b(\beta)}p_c(\alpha)\right)\right]$$

if state β occurs and

$$v_h[p_c(\alpha), 0]$$

if state α occurs.

Finally, if agent h in G_0 does not trade in assets, the utilities she gains are those associated with the two deterministic equilibria: $U_h[x_h^*(\alpha)]$ if α occurs and $U_h[x_h^*(\beta)]$ if β occurs.

It is now possible to define the expression $v_h^{s(\max)} - v_h^s$ as the generic regret associated with an amount $b_h(\cdot)$ of Arrow security for agent h when state s has occurred. In a general equilibrium framework the optimization under uncertainty via the MMRC requires that all the regrets be equalized across all states of nature. With only two states the following 'optimum condition' must hold[5]:

$$v_h^{\alpha(\max)} - v_h^{\alpha} = v_h^{\beta(\max)} - v_h^{\beta} \tag{3}$$

Analogously the 'optimum condition' under MMC requires that the minima be directly equalized across all states of nature. With only two states, the condition is:

$$v_h^{\alpha} = v_h^{\beta} \tag{4}$$

We can now state the two following propositions.

Proposition 2 *If agents in G_0 make use of the minimax regret criterion, a unique sunspot equilibrium exists in the economy. The vector of the equilibrium prices of the Arrow securities $[p_b^*(\alpha); p_b^*(\beta)]$*

[5] Just to give an intution suppose that, for a given price, agent h has bought an amount of Arrow security $\tilde{b}_h(\beta) > 0$ such that $v_h^{\alpha(\max)} - \tilde{v}_h^{\alpha} > v_h^{\beta(\max)} - \tilde{v}_h^{\beta}$. Since what matters under the MMRC is the *maximum* regret across the states, in this situation the agent would find it profitable to start selling that security until the two regrets would converge towards each other. Only when (3) holds exactly, there is no more incentive to trade in assets, since the maximum regret is at its minimum.

is such that no trade in Arrow securities takes place in equilibrium, i.e. $b_h(s) = 0$. *Moreover, the prices of the* l *commodities,* $[p_c^*(\alpha); p_c^*(\beta)]$, *and the consumption allocations,* $x_h^* = [x_h^*(\alpha); x_h^*(\beta)]$ *for* h *in* G_0, G_1, *are those corresponding to the two Walrasian equilibria.*

Proof. We prove our result in two stages. In the first we show that, if an equilibrium exists, it must be characterized by no trade in Arrow securities. In the second this equilibrium is pinned down and the equilibrium prices of Arrow securities are found.

1. By definition of Arrow securities, an equilibrium must be characterized by:

$$\sum_{h \in G_0} b_h(s) = 0 \text{ for } s = \alpha, \beta \tag{5}$$

Moreover, since agents in G_0 are identical, then in equilibrium all individuals choose the same unique optimal portfolio. Hence:

$$b_h(s) = \bar{b}(s) \quad \forall h \text{ in } G_0. \tag{6}$$

Eq.s (5) and (6) imply $b_h(s) = 0$. Then, if an equilibrium exists, it must be characterized by no trade in Arrow securities.

2. Indeed, the unique consumption allocation compatible with no trade in Arrow securities is the pair $x_h^* = [x_h^*(\alpha); x_h^*(\beta)]$. Now we prove that a vector of Arrow securities' prices $p_b^* = [p_b^*(\alpha); p_b^*(\beta)]$ always exists, which renders the vectors of prices $p^* = [p_c^*(\alpha); p_c^*(\beta); p_b^*(\alpha); p_b^*(\beta)]$ and of allocations $x_h^* = [x_h^*(\alpha); x_h^*(\beta)]$ for h in G_0, G_1 the unique sunspot equilibrium of our economy.

Since agents apply the MMRC, and since in equilibrium it holds $p_b(\alpha) + p_b(\beta) = 1$, Arrow securities' prices are determined via the following system:

$$\begin{cases} v_h^{\alpha(\max)} - U_h[x_h^*(\alpha)] = v_h^{\beta(\max)} - U_h[x_h^*(\beta)] \\ \\ p_b(\alpha) + p_b(\beta) = 1 \end{cases} \tag{7}$$

The first equation of system (7) equalizes the regret associated with the consumption bundle $x^*(\alpha)$ to the one associated with $x^*(\beta)$. It is a special case of the 'optimum condition' under MMRC (equation (3)), obtained when $b_h(s) = 0$.

Continuity and monotonicity of the utility function $U_h(\cdot)$, for h in G_0, constitute sufficient conditions for the existence of a solution $0 < p_b^*(s) < 1$, for $s = \alpha, \beta$, in system (7). In fact (for a graphical intuition of this result see figure 1):

$$\lim_{p_b(\alpha)\to 0} v_h^{\alpha(\max)} - U_h[x_h^*(\alpha)] > 0; \lim_{p_b(\alpha)\to 0} v_h^{\beta(\max)} - U_h[x_h^*(\beta)] = 0$$

and:

$$\lim_{p_b(\alpha)\to 1} v_h^{\alpha(\max)} - U_h[x_h^*(\alpha)] = 0; \lim_{p_b(\alpha)\to 1} v_h^{\beta(\max)} - U_h[x_h^*(\beta)] > 0$$

Since it generically holds $x_h^*(\alpha) \neq x_h^*(\beta)$, the equilibrium is characterized by sunspot activity. ■

Proposition 3 *If agents make use of the maxmin return criterion, then sunspots do not matter.*

Proof. The first part of the proof is exactly the same as the one in the previous proposition, in which we have shown that in equilibrium no trade in Arrow securities can take place ($b_h(s) = 0$). However, since in general $U[x_h^*(\alpha)] \neq U[x_h^*(\beta)]$, $x^* = [x^*(\alpha); x^*(\beta)]$ cannot be the optimal solution for this decision rule (recall equation (4)). In fact, for every vector of asset prices p_b', G_0-agents would be better off by buying an amount of securities $b_h'(\alpha) \neq 0$ such that:

$$\begin{cases} v_h\left[p_c(\alpha), p_c(\alpha)\omega_h + b_h'(\alpha)\right] = v_h\left[p_c(\beta), p_c(\beta)\omega_h - b_h'(\alpha)\dfrac{p_b'(\alpha)}{p_b'(\beta)}\right] \\ p_b(\alpha) + p_b(\beta) = 1 \end{cases} \tag{8}$$

This configuration is however not sustainable in equilibrium, since it would imply trade in asset markets, while it must necessarily be $b_h^*(s) = 0$ $\forall h$ in G_0. Hence a sunspot equilibrium does not exist. ■

Chapter 2

Uncertainty Averse Bank Runners

2.1 Introduction

Starting from the seminal Diamond and Dybvig's (1983) paper (D-D henceforth), a stream of literature has developed which looks at bank runs as phenomena originating from a coordination failure driven by an extrinsic random variable, namely a 'sunspot'. One of the most recent and important contributions to the topic is Peck and Shell (2003) which, along the lines of the 'classical' D-D, designs a model admitting a multiplicity of equilibria and further develops the issue of the selection among them. A significant departure of this framework from D-D, tracing back to Wallace (1988), is the broadening of the set of feasible contractual arrangements from the 'simple contracting' (so-called by Green and Lin (2000)) considered by D-D to a class of banking mechanisms that allow for suspension schemes.

In what follows we will refer in particular to Peck and Shell (2003), whose model is modified

[0] This chapter is drawn from a joint paper with Guido Cozzi.

in order to encompass a depositor with *uncertain beliefs* on her position in the queue *in the case a bank run occurs*. Uncertainty is to be intended in the sense, first given by Knight (1921), that the information of each depositor is too vague to be represented by an objective probability distribution. It is then assumed that, when each depositor expects a (rarely observed) run to occur, she is no longer able to evaluate correctly the probability distribution of her position number in the queue. The *rationale* for introducing such an assumption can be justified as follows: since the bank, in finding the optimal contract, is allowed to assign different pay-offs across depositors as a function of their place in line, when a run occurs each depositor might feel penalized, in terms of insufficient information about her personal 'running skills' with respect to her 'competitors', to gain the relatively highest pay-offs. In particular, if partial suspension of convertibility characterizes the optimal mechanism (Wallace (1990)), the patient depositor might reasonably be afraid of being located among the last positions in the queue and, as we will clarify below, might be eventually discouraged from running.

Our formalization of the depositor's attitude towards uncertainty is inspired by the (multi-prior) maximin expected utility (MEU) theory axiomatized by Gilboa and Schmeidler (1989) (see in particular Subsection 0.3.2). The application of this decision rule to our framework leads to assume a depositor who maximizes her expected pay-off with respect to the binary choice -whether or not to withdraw -, while selecting the worst probability distribution (over her position in the queue) among all the admissible ones. As we will show in the next Section, since in a mechanism design approach pay-offs generally vary as a function of the position number, uncertainty aversion may alter the agent's withdrawal strategy. Our proposition states that, in a MEU approach to decision-making under uncertainty, there always exists a set of minimizing prior beliefs which makes the bank run equilibrium disappear. Interestingly, this result is obtained *independently* of the bank's solution to the problem of choosing the optimal mechanism[1]. Consequently we suggest that 'suspension schemes' are worthy, not only because they improve risk-sharing (Wallace (1990)), but also because they may undermine panic-driven bank runs in a potentially general class of frameworks.

2.2 Aversion to Uncertainty and Propensity to Run

The banking model developed in Peck and Shell (2003) is characterized by aggregate uncertainty on the distribution of the agent's type and by the observance of the so-called sequential service constraint (which forces the bank to deal with customers sequentially). There are three periods, and N potential depositors, α being the number of impatients and $N-\alpha$ that of patients. Each of them is endowed with y units of consumption in period 0 regardless of type. Impatient agents evaluate utility of period 1 only, through a function $u(c^1)$, while patient agents, who are allowed to costlessly store consumption across periods, evaluate utility of both periods 1 and 2 through the function $v(c^1+c^2)$, where c^1 and c^2 represent respectively consumption received in periods 1 and 2. Both functions are assumed to be strictly increasing, concave, and twice continuously differentiable. The bank, whose target is to maximize the ex-ante expected utility of consumers, knows the probability distribution over all possible realizations of types [$f(\alpha)$ for $\alpha = 0, 1, ..., N$] and, as usual, is not able to recognize the agent's type. As far as technology is concerned, 1 unit of consumption invested in period 0 yields R units in period 2 and 1 unit in period 1. As a consequence of the technology and preference assumptions, in autarchy patient depositors strictly prefer to consume in period 2.

In Peck and Shell (2003) an essential distinction is made between pre- and post-deposit game. In the latter consumers are assumed to have already deposited their endowments and, after having

[1] Notice that the bank's problem of finding the optimal contract slightly changes when uncertainty aversion is introduced. The bank must in fact take into account this uncertainty aversion in the incentive compatibility constraint when all depositors are impatient. The validity of our proposition is however not affected by this change.

learnt their type (at the beginning of period 1), must only decide whether to withdraw in period 1 or in period 2. The pre-deposit game also encompasses the agent's choice between deposit and autarchy: this choice is indeed not trivial since, for example, the agent would decide not to deposit if she knew that a bank run would occur with probability 1. Here we are only concerned with the post-deposit game.

The best solution to the post-deposit game in Peck and Shell (2003) is obtained by maximizing total welfare, defined as the sum of the utilities of the two types weighted with the probabilities of all possible realizations, subject to the resource constraint and to an incentive compatibility constraint (ICC) stating that patient depositors, in comparing the expected pay-off associated with the 'truth telling' strategy (withdrawing in period 2) with that associated with the strategy of 'lying' (withdrawing in period 1), must prefer to tell the truth. The solution to the problem reveals that, even if the ICC holds, the economy may be subject to a bank run. The no-bankrun condition (NBC) that would be violated in this case can be written as follows:

$$\frac{1}{N}\sum_{z=1}^{N} v(c^1(z)) \leq v\left(\left[Ny - \sum_{z=1}^{N-1} c^1(z)\right] R\right) \tag{NBC}$$

where z refers to the depositor's position in the queue. This condition states that, even though the patient depositor had the belief that any other agent would be running, she would be however interested in waiting until period 2.

Our departure from this framework is concerned with the probability distribution with which each agent is assumed to be endowed in order to evaluate her position in the queue. In Peck and Shell (2003) the agent evaluates each place in line as equally likely independently of whether or not a bank run is expected. Conversely, for the reasons stated in the introduction we allow probabilities to vary across position numbers whenever depositors believe that such an unusual event as a run is about to occur. Following the Gilboa and Schmeidler's (1989) MEU theory, we further assume that, when a bank run is expected:

1. the agent's subjective belief about her own position in the queue is modeled as a set of additive probability measures (*multiple prior belief);*

2. the agent's choice behavior is represented as a maxmin strategy, which drives her to maximize her utility with respect to a binary choice - whether to withdraw in period 1 or 2 - and, at the same time, to find the additive probability distribution (over her position number) which minimizes the pay-off associated with withdrawing in period 1. We can now state the following:

Proposition 4 *For the post-deposit game there exists a positive measure set of minimizing priors which makes the bank run equilibrium disappear.*

Proof. In the NBC the standard probability distribution over all positions in the queue can be described as:

$$q_z = \frac{1}{N} \; \forall z = 1, ..., N$$

where z stands for the position and q_z for the probability of being the z-th in the queue. Now replace it with the following set of priors:

$$\tilde{q}_z = [0 + \varepsilon, 1 - \varepsilon] \text{ for } \varepsilon > 0 \text{ and } \forall z = 1, ..., N, \tag{1}$$

and suppose -w.l.o.g., as it will be argued below - that 'weak' PSC characterizes the optimal solution:

$$c^1(1) \geq c^1(2) \geq ... \geq c^1(N-1) \geq Ny - \sum_{z=1}^{N-1} c_1(z).$$

The relation above identifies two possible cases:

1. The optimal solution is:

$$c^1(1) = c^1(2) = ... = c^1(N-1) = Ny - \sum_{z=1}^{N-1} c_1(z) \tag{2}$$

In this case the minimizing distribution is anyone among all possible additive distributions belonging to the set defined in (1). Then the NBC becomes:

$$v\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)<v\left[\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)R\right],$$

which is always satisfied $\forall R>0$ and no bank run can occur. Notice that (2) corresponds to the 'autarchic solution'.

2. In the optimal solution, at least one pay-off is strictly greater than the others. Suppose (w. l. o. g.) that:

$$c^1(1)\geq c^1(2)\geq ...\geq c^1(N-1)>Ny-\sum_{z=1}^{N-1}c_1(z).$$

In this case the minimizing prior with respect to (1) would be:

$$[q_z=\varepsilon\ \forall z=1,...,N-1;q_N=1-(N-1)\varepsilon]$$

and the NBC becomes:

$$\sum_{z=1}^{N-1}\varepsilon v(c^1(z))+[1-(N-1)\varepsilon]\,v\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)<v\left[\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)R\right]$$

We argue that, $\forall R>0$, there is at least an $\varepsilon>0$ that satisfies the condition stated above. The threshold value of ε below which the bank run disappears is:

$$0<\varepsilon=\frac{v\left[\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)R\right]-v\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)}{\sum_{z=1}^{N-1}v(c^1(z))-(N-1)v\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)}$$

Notice also that the assumption of PSC has been made w.l.o.g. Indeed suppose that the pay-off associated with the last position is not the minimum because there exists:

$$v(c^1(i))<v\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)\text{ for some }i\in[1,N-1];$$

then it will also be:

$$v(c^1(i))<v\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)<v\left[\left(Ny-\sum_{z=1}^{N-1}c_1(z)\right)R\right]$$

and the reasoning of the proof can be repeated identically. ■

Part II

Strong Uncertainty in Neo-Schumpeterian Growth Theory

"He [the entrepreneur] has the capacity to see things in a way, which afterwards proves to be true, although it cannot be proven at the time" *(Joseph Alois Schumpeter, 1934)*

Chapter 3

Is Strong Uncertainty Harmful for Schumpeterian Growth?

3.1 Introduction

In Schumpeterian growth theory (SGT) the Schumpeter's view of economic development, as spurred by incessant R&D races aimed at gaining monopoly profits, is incorporated into an Arrow-Debreu dynamic general equilibrium framework with 'measurable uncertainty' (risk). The assumption of a perfectly assessable investment horizon - that is, the idea that transparent and well-organized financial markets allow savers to finance R&D activity in the light of an expected discounted value of future returns 'revealed' by an efficient stock market - is standard along growth models, such as Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Segerstrom (1998),

[0] This chapter is drawn from a joint paper with Guido Cozzi.

Howitt (1999), Aghion et al. (2001).

In this chapter we claim that R&D investment decisions are actually taken under conditions of strict uncertainty on their possible returns. We then remove the assumption of a 'rigorously calculable future' and provide a first attempt to introduce formally strict uncertainty on the process describing the evolution of innovation. This idea is close to the Schumpeter's original view of the innovation process as a process brought about by 'resourceful' entrepreneurs and characterized by some intrinsically unpredictable aspect (Schumpeter (1934) and (1939)).

According to SGT the arrival of innovation in the economy follows a Poisson process. Also, the parameter of this process, representing the 'flow probability' of innovation, is perfectly known by the R&D investment decision-maker (DM). In particular, in the framework developed by Aghion and Howitt (1992), the value of this parameter named λ, affects both the problem of whether or not to invest in R&D (whose solution is embodied by the 'arbitrage equation'), and the problem of whether to invest in risk-free assets or in shares of monopolistic firms (whose solution is embodied by the 'asset equation').

In this framework we assume that investors know neither the exact value of the parameter λ nor a prior distribution over the set of its potential values. Hence, they face the two decision problems sketched out above under conditions of complete ignorance and are assumed to find their optimal choice in each of them via four alternative decision criteria (see Luce and Raiffa (1958) and, for a brief review see chapter 0 of this book): the 'maximin return criterion' (MMC), the 'minimax regret criterion' (MMRC), the 'optimism-pessimism index criterion' (OPIC) and the Laplace's 'principle of insufficient reason' (PIR). The equilibrium R&D efforts in steady-state are then determined under all these decision rules, and compared with each other and with the standard solution of Aghion and Howitt. In particular, we prove that the same amount of R&D investments is carried out under

MMRC and PIR. Also, this amount is higher than the one associated with MMC. Finally, R&D investments under OPIC can never be lower than those carried out under MMC.

This chapter represents a firts attempt to introduce formally, into a standard Schumpeterian framework, investors' ignorance about the returns associated with their R&D investments. Ignorance is here represented as lack of information about the exact arrival rate of innovation.

The rest of the chapter is organized as follows. In the next Section we provide a reminder of the model of Aghion and Howitt (1992). In Section 3.3 we apply the four standard decision rules under complete ignorance to the R&D investment decision problems, determine the steady-state equilibrium R&D efforts under each of them and provide comparative statics analysis. In Section 3.4 we provide the welfare analysis, while in Section 3.5 we conclude with some remarks.

3.2 A Reminder of the Model

In this Section we briefly recall the basic framework developed in Aghion and Howitt (1992). Time is continuous and there exists a continuum of infinitely lived households with identical intertemporally additive preferences, with r representing the rate of time preference. Since instantaneous utility is assumed to be linear and there are perfect capital markets, then r also turns out to be the equilibrium interest rate. Households are endowed with flow units of skilled or unskilled labor time and are assumed to supply them inelastically in a perfectly competitive market.

There is a perfectly competitive final sector, in which output is produced according to a constant returns to scale (CRS) technology. For simplicity, we assume a Cobb-Douglas specification:

$$y_t = A_t x_t^\theta N_t^{1-\theta} = A_t x_t^\theta \qquad 0 < \theta < 1$$

where y is final output, x is the intermediate good and N, normalized to 1, is the unskilled labor. A is the productivity parameter, which is assumed to evolve according to the following rule:

$A_{t+1} = \gamma A_t$ for $\gamma > 1$ and $t = 0, 1, 2...$

The subscript t does not refer to calendar time but to the generation of the intermediate product that is being used. Whenever a new intermediate product is introduced into the market, the economy jumps of γ. The intermediate good x is produced through a one-to-one technology from skilled labor (L). The price of the final output is assumed as the numerarie: $p(y_t) = 1$.

Before describing the innovation process, let us illustrate what happens when a new quality is discovered: as soon as a new intermediate product is introduced, it is automatically protected by a perfect and infinitely lived patent, which allows the inventor (or whoever buys the blueprint) to temporarily monopolize the market. With the assumption that innovations are drastic, monopoly profits can be easily obtained from the profit maximizing condition:

$$\max_{x_t} \left[\pi_t = A_t \theta x_t^{\theta-1} x_t - x_t w_t\right]$$

where w_t is the skilled labor wage. This maximization gives the optimal value of x_t as:

$$x_t = \left(\frac{w_t}{A_t \theta^2}\right)^{\frac{1}{1-\theta}}.$$

Maximum profits can then be written as:

$$\pi_t = \frac{1-\theta}{\theta} x_t w_t. \qquad (1)$$

The innovation process takes place because R&D firms employ, in a perfectly competitive market, an amount n of skilled labor in order to gain a probability of discovering the next vintage. Since skilled labor can switch from the research sector to the intermediate sector and viceversa, the skilled labor market clearing condition can be written as:

$$L = x_t + n_t$$

where x_t and n_t represent labor employed respectively in the intermediate and the research sectors. We also define V_t as the market value of the monopolistic firm producing vintage t.

According to the standard Schumpeterian literature, the arrival of innovation in the economy is assumed to follow a Poisson Process. The parameter λ of this process, representing the flow probability of an innovation, is known by the investor. Because of the CRS in the research sector, the number of R&D firms is indeterminate. In equilibrium expected benefits from a unit of R&D effort (λV_{t+1}) must equal its cost (w_t). The equation

$$\lambda V_{t+1} = w_t \tag{2}$$

is usually called the 'research arbitrage equation' of the model. Furthermore, because instantaneous utilities are linear, agents must be indifferent between investing in shares of the incumbents and investing in risk-free assets. Then the value V_t must satisfy the following 'asset equation':

$$rV_t = \pi_t - \lambda n_t V_t$$

where rV_t is the return from investing in risk-free shares, π_t is the flow of profits corresponding to vintage t, while $\lambda n_t V_t$ is the expected capital loss due to the introduction of vintage $t+1$ and embodies the Schumpeter's idea of the 'cretive destruction effect' of any innovation. The asset equation gives the expression for V_t as:

$$V_t = \frac{\pi_t}{r + \lambda n_t}, \tag{3}$$

stating that the market value of the monopolist firm is the flow of profits that it will produce, discounted at the obsolescence-adjusted interest rate. We are now ready to modify this basic set-up so as to incorporate the idea that investors are completely ignorant about the arrival rate of innovation.

3.3 The New Equilibrium R&D Efforts under Complete Ignorance

Assume that the parameter λ is strictly uncertain, in the sense that the investors know neither its exact value nor a probability distribution over all possible values it can take. The unique assumption we pose on the investors' knowledge is that they consider some 'particularly low', as well as some 'particularly high' values of λ, as unreasonable and, hence, that they can exclude them from their decision problem. In other words, we require decision makers to know the set of states of nature which potentially occur, and to ignore totally the probability of realization of each of them. We then state the following assumption, which will hold throughout.

Assumption. *Investors exclude all values of λ below some lower threshold $m \in]0, \infty[$ and all values above some upper threshold $M > m$; that is to say, they believe that $\lambda \in [m, M]$. Moreover, they are not provided with any probability distribution over this set of values.*

In the light of this assumption, two decision problems stated in Section 3 must be reconsidered. The former is the problem of whether or not to devote investments in R&D and the latter is the problem of whether to invest in shares of the incumbents or in risk-free assets. We will study them in order under the four criteria introduced in Section 2.

3.3.1 Problem 1 (The Arbitrage Equation)

Assume that the economy is in t[1]. The DM has to decide whether or not to invest in R&D by comparing the profitability associated with these two alternatives, as the parameter λ varies from m to M. Then there are two possible acts, 'R&D investment' and 'no R&D investment', and the set

[1] That is, assume that generation t of the intermediate good is being produced.

of states of nature is the interval $[m, M]$. The pay-offs associated with each pair act/state of nature are as follows. If the investor does not carry out any R&D investment, her pay-off will always be null, independently of the true state of nature. If she does, the cost of each R&D investment unit is always the skilled labor wage (w_t), while expected benefits (λV_{t+1}) depend on the strictly uncertain parameter $\lambda \in [m, M]$. Hence she reckons that, for each unit of skilled labor employed, her pay-off from R&D investment is $\lambda V_{t+1} - w_t$ for[2] $\lambda \in [m, M]$.

Given this decision problem, let us first assume that, whenever facing non-probabilistic uncertainty, the DM makes use of the maximin return criterion in order to find her optimal choice. Then, for each act, the DM only takes into account the pay-off associated with the worst state inside the set $[m, M]$, and selects the act for which this pay-off is maximum. Hence, she compares a null pay-off (associated with the act 'no R&D investment') with the amount $mV_{t+1} - w_t$, representing the worst pay-off associated with the act 'R&D investment', and corresponding to the state $\lambda = m$. If $mV_{t+1} - w_t < 0$, she will not invest in R&D, while if $mV_{t+1} - w_t > 0$, then R&D investment will be infinite. Equilibrium is reached when the arbitrage condition $mV_{t+1} - w_t = 0$ holds. This condition, which identifies a situation of indifference as to whether or not to invest in R&D, can equivalently be written as:

$$w_t = mV_{t+1}. \tag{4}$$

Alternatively, if the DM makes her optimal choice via the minimax regret criterion, then her purpose is to choose the act which minimizes the maximum regret for $\lambda \in [m, M]$. As stated by the

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[2] Clearly λ is assumed to take all real values in the interval $[m, M]$ but, for the sake of simplicity, assume for a moment that λ can only take n values between m and M, and that $\lambda_1 = m$ and $\lambda_n = M$. The decision problem can now be formalized via the following matrix:

Acts/States	$\lambda_1 = m$	λ_2	...	λ_i	...	$\lambda_n = M$
R&D	$mV_{t+1} - w_t$	$\lambda_2 V_{t+1} - w_t$	...	$\lambda_i V_{t+1} - w_t$	...	$MV_{t+1} - w_t$
No R&D	0	0	...	0	...	0

where the values of λ have been arranged in increasing order: $\lambda_1 < \lambda_2 < ... < \lambda_i < ... < \lambda_n$.

This formalization with discrete states of nature has only been provided to give a simple intuition of the decision problem we have been illustrating.

MMRC (see Section 2), for each act the investor only takes into account the pay-off associated with the state for which the regret is maximized. The maximum regret associated with the act 'R&D investment' is $w_t - mV_{t+1}$ and is attained when $\lambda = m$, while the maximum regret associated with the act 'no R&D investment' is $MV_{t+1} - w_t$ and is attained when $\lambda = M$. Afterwards she compares these maximum regrets and eventually picks up the act associated with the smallest one. Hence, if $w_t - mV_{t+1} > MV_{t+1} - w_t$, she will not carry out R&D, while if $w_t - mV_{t+1} < MV_{t+1} - w_t$, then R&D investment will be infinite. In equilibrium the arbitrage condition $w_t - mV_{t+1} = MV_{t+1} - w_t$ must hold, which can also be written as:

$$w_t = \frac{m+M}{2} V_{t+1}. \tag{5}$$

Remark 5 *Notice that, with the minimax regret criterion, all intermediate states of nature (that is, all $\lambda \in\,]m, M[$) turn out to be irrelevant, since they do never identify a situation of maximum regret for neither of the two acts. Maximum regret is instead identified by the two extreme cases of $\lambda = m$ and $\lambda = M$.*

If the optimism-pessimism index is the criterion adopted, the DM evaluates her expected returns from the act 'R&D investment' by computing a α-weighted average of the security level $(mV_{t+1} - w_t)$ and the optimism level $(MV_{t+1} - w_t)$, that is, $[\alpha m + (1-\alpha)M]\, V_{t+1} - w_t$, where $0 \leq \alpha \leq 1$ is a parameter roughly measuring her 'degree of pessimism'. By comparing this pay-off with the null pay-off associated with 'no R&D investment', indifference as to whether or not to invest in R&D can then be expressed via the following arbitrage equation:

$$w_t = [\alpha m + (1-\alpha)M]\, V_{t+1}. \tag{6}$$

Finally, if the principle of insufficient reason guides the investor's choice, then this investor considers every state $\lambda \in [m, M]$ as equally likely and, hence, her expected benefits from investing in R&D to be equal to[3] $\frac{m+M}{2}V_{t+1} - w_t$. As under MMRC, the arbitrage equation under PIR is then given by equation (5).

3.3.2 Problem 2 (The Market Value of Incumbents)

As in Aghion and Howitt (1992), in order to derive the expression for V_t, we have to address the agent's problem of whether to invest in risk-free assets or in shares of current monopolists. This decision problem must also be revised in order to take into account the strict uncertainty associated with the parameter λ and the decision rule which is adopted in the face of uncertainty. There are two possible acts, investing in risk-free assets or in shares of the monopolistic firms, and an infinite and bounded set of states of nature $\lambda \in [m, M]$. If the investor decides to buy risk-free assets, her return will always be rV_t, independently of the productivity of the research technology. On the other hand, if she invests in shares of the incumbents, then her pay-off will be $\pi_t - \lambda n_t V_t$, which clearly depends on the uncertain parameter[4] $\lambda \in [m, M]$. Notice that, since the parameter λ represents the productivity of the R&D aimed at discovering vintage $t+1$, the risky asset return is a decreasing function of λ.

An investor adopting the MMC compares the worst pay-offs associated with each act, and eventually selects the act corresponding to the best one. In comparing the worst pay-offs, rV_t for choosing risk-free assets and $\pi_t - Mn_tV_t$ for choosing the incumbent's shares, indifference as to whether to invest in shares or in risk-free assets is reached when these values equalize[5]:

[3] Expected benefits are obtained via the following integral: $\frac{1}{M-m}\int_m^M \lambda V_{t+1} d\lambda - w_t = \frac{m+M}{2}V_{t+1} - w_t$.

[4] As before, for the sake of simplicity we can imagine that λ takes a finite set of values, arranged in increasing order: $\lambda_1 = m < \lambda_2 < ... < \lambda_i < ... < \lambda_n = M$,
and formalize the decision problem via the following matrix:

Acts/States	$\lambda_1 = m$	λ_2	...	λ_i	...	$\lambda_n = M$
Shares	$\pi_t - mn_tV_t$	$\pi_t - \lambda_2 n_tV_t$	...	$\pi_t - \lambda_i n_tV_t$	...	$\pi_t - Mn_tV_t$
Risk-Free Assets	rV_t	rV_t	...	rV_t	...	rV_t

[5] Remember that, by assumption, the DM is risk neutral.

$rV_t = \pi_t - Mn_tV_t.$

From the expression above we determine the market value of the monopolistic firm producing vintage t as:

$$V_t = \frac{\pi_t}{r + Mn_t} \tag{7}$$

If the investor makes use of the MMRC, then for each act she compares the two maximum regrets, as the state of nature λ varies from m to M, and singles out the act for which this value is minimum. If she decides to invest in risk-free assets, then her maximum regret is $\pi_t - mn_tV_t - rV_t$, which is associated with the state $\lambda = m$, that is, a minimal 'creative destruction effect'. On the other hand, if she decides to invest in shares, her maximum regret is $rV_t - (\pi_t - Mn_tV_t)$, which is associated with $\lambda = M$: in this case, the 'creative destruction effect' is at its maximum. In equilibrium it must be:

$$\pi_t - mn_tV_t - rV_t = rV_t - \pi_t + Mn_tV_t,$$

which gives the following expression for V_t:

$$V_t = \frac{\pi_t}{r + \frac{m+M}{2}n_t}. \tag{8}$$

The same consideration made in remark 1 holds true here.

When the OPIC is the investor's decision criterion, the pay-off associated with investing in shares is given by the α-weighted average of the security level $(\pi_t - Mn_tV_t)$ and the optimism level[6] $(\pi_t - mn_tV_t)$, while the one corresponding to investing in risk-free assets is always rV_t. Then in equilibrium it must be:

$$rV_t = \pi_t - [\alpha M + (1-\alpha)m]n_tV_t,$$

and hence:

[6] Notice that, as opposite to problem 1, now m and M are respectively associated with the optimism level and the security level.

$$V_t = \frac{\pi_t}{r + [\alpha M + (1-\alpha)m]n_t}. \quad (9)$$

Finally, when the principle of insufficient reason is used, then investing in shares brings about expected returns equal to $\pi_t - \frac{m+M}{2} n_t V_t$, while investing in risk-free assets always brings about rV_t. As in the case of MMRC, equilibrium is then described by equation (8).

3.3.3 The Steady-State Equilibrium

In this Subsection we focus on the steady-state equilibrium of the model, determine and compare the equilibrium R&D efforts under all the four decision criteria introduced above. Consider first the case of the maximin return criterion. The market value of the monopolistic firm producing vintage $t+1$ is:

$$V_{t+1} = \frac{\pi_{t+1}}{r + Mn_{t+1}},$$

where

$$\pi_{t+1} = \gamma \pi_t = \gamma \frac{1-\theta}{\theta} x_t w_t,$$

After some manipulations[7] we obtain the system, composed of the arbitrage equation and the labor market-clearing condition, which describes the evolution of this economy:

$$\begin{cases} w_t = m \dfrac{\gamma \frac{1-\theta}{\theta} x_t w_t}{r + Mn_{t+1}} \\ L = x_t + n_t \end{cases}$$

Turning to the steady-state, where $n_t = n_{t+1}$, we can rewrite this system as:

$$\begin{cases} w = m \dfrac{\gamma \frac{1-\theta}{\theta} xw}{r + Mn} \\ L = x + n \end{cases} \quad (10)$$

from which we can easily determine the equilibrium value of the research effort:

$$n^* = \frac{\gamma m \frac{1-\theta}{\theta} L - r}{M + \gamma m \frac{1-\theta}{\theta}}.$$

[7] We have substituted for V_{t+1} given above into (4).

We can proceed exactly through the same steps in order to obtain the R&D efforts associated with the three other decision rules. Under the MMRC, we have:

$$V_{t+1} = \frac{\pi_{t+1}}{r + \frac{m+M}{2} n_{t+1}},$$

and, substituting for π_{t+1}, we can write the system describing the evolution of this economy as:

$$\begin{cases} w_t = \frac{m+M}{2} \dfrac{\gamma \frac{1-\theta}{\theta} x_t w_t}{r + \frac{m+M}{2} n_{t+1}} \\ \qquad L = x_t + n_t \end{cases}.$$

In steady-state, where $n_t = n_{t+1}$, this system becomes:

$$\begin{cases} w = \frac{m+M}{2} \dfrac{\gamma \frac{1-\theta}{\theta} x w}{r + \frac{m+M}{2} n} \\ \qquad L = x + n \end{cases} \tag{11}$$

The equilibrium value of the research effort under the MMRC is then:

$$n^{**} = \frac{\gamma \frac{m+M}{2} \frac{1-\theta}{\theta} L - r}{\frac{m+M}{2} + \gamma \frac{m+M}{2} \frac{1-\theta}{\theta}}.$$

Under the optimism-pessimism index criterion, the steady-state system is instead as follows:

$$\begin{cases} w = [\alpha m + (1-\alpha) M] \dfrac{\gamma \frac{1-\theta}{\theta} x w}{r + [\alpha M + (1-\alpha) m] n} \\ \qquad L = x + n \end{cases} \tag{12}$$

and gives the following equilibrium R&D effort:

$$n^{***} = \frac{[\alpha m + (1-\alpha) M] \gamma \frac{1-\theta}{\theta} L - r}{[\alpha M + (1-\alpha) m] + \gamma [\alpha m + (1-\alpha) M] \frac{1-\theta}{\theta}}.$$

Finally, under the principle of insufficient reason, the steady-state system coincides with system (11) and, hence, the equilibrium R&D efforts are:

$$n^{****} \equiv n^{**} = \frac{\gamma \frac{m+M}{2} \frac{1-\alpha}{\alpha} L - r}{\frac{m+M}{2} + \gamma \frac{m+M}{2} \frac{1-\alpha}{\alpha}}$$

First notice that $n^{**} > n^*$, that is, the equilibrium R&D investments are higher under the MMRC (or PIR) than under the MMC. Even though it is not apparent by looking at the two expressions for n^* and n^{**}, this result can be easily verified via the following equations, derived respectively from systems (10) and (11):

$$1 = \frac{m \gamma \frac{1-\alpha}{\alpha} (L - n^*)}{r + M n^*} \tag{13}$$

$$1 = \frac{\frac{m+M}{2}\gamma\frac{1-\alpha}{\alpha}(L-n^{**})}{r+\frac{m+M}{2}n^{**}} \tag{14}$$

If n^* were equal to n^{**}, then we could unambiguosly conclude that the right-hand side of (13) would be lower than the right-hand side of (14) (given $m < M$). Yet, since they both must equal 1, and since both the right-hand sides are decreasing functions of n, it must be $n^{**} > n^*$.

Moreover, from the definition of the MMC and OPIC, it follows trivially: $n^{***} > n^* \ \forall\ 0 \leq \alpha < 1$ and $n^{***} = n^*$ if and only $\alpha = 1$, that is, if the degree of pessimism is maximum. Obviously, whether $n^{***} \lesseqgtr n^{**}$ crucially depends on the parameter α.

Finally all these values closely resemble the one found by Aghion and Howitt (1992) in their Cobb-Douglas example:

$$n^{(AH)} = \frac{\gamma\lambda\frac{1-\theta}{\theta}L - r}{\lambda + \gamma\lambda\frac{1-\theta}{\theta}},$$

the difference lying in the fact that, in all our equilibrium solutions, some function of m and/or of M replaces the (unknown) parameter λ. In particular, in $n^{**}(= n^{****})$ the expression $\frac{m+M}{2}$ exactly replaces λ: hence, if investors are uncertain about the arrival rate of innovation, and if they face non-probabilistic uncertainty via MMRC (or PIR), these investors act *as if* $\lambda = \frac{m+M}{2}$.

Comparative statics analysis for γ, L, θ and r is perfectly in line with the Aghion-Howitt's (1992) results: both a higher quality jump γ and a larger amount of skilled labor force L raise the equilibrium R&D effort n (under whichever criterion it is determined), while a higher rate of interest r, and a higher value of θ (inversely measuring the degree of market power) lower it. The relationship between the arrival rate of innovation and n is instead less immediate. As in the Aghion-Howitt's (1992) model there are two conflicting effects. On the one hand, an increase in the arrival rate makes the research activity more productive for a given level of employment, thus stimulating the R&D effort. On the other hand, this increase exacerbates the creative destruction effect, reducing the R&D effort. While in Aghion-Howitt (1992), the former effect dominates the latter (making then

$n^{(AH)}$ be a positive function of λ), the whole effect here depends on the decision rule the investors are assumed to adopt. If m and/or M increase, then $n^{**}(= n^{****})$ will unambiguosly increase, since what ultimately matters in its expression is the average value $\frac{m+M}{2}$. The same relationship does not hold for n^{*} (and by continuity for n^{***}): it is easy to show that n^{*} is an increasing function of m (which is responsible for the positive 'productivity effect'), and a decreasing function of M (which is responsible for the negative 'creative destruction effect').

3.4 Welfare Analysis

In this Section we compare, for each decision rule, the laissez-faire equilibrium R&D effort with the one chosen by a social planner seeking to maximize the welfare of the representative agent. Such welfare, called U_t, is the valuation, based on the risk free rate of time preference r, of the consumption available at all future dates. The reasoning underlying the derivation of U_t closely resembles the one carried out to derive V_t in (3), with two important differererences: first, as the reader recalls from Section 3, in determining the market value of the monopolistic firm we had to bear in mind that the shareholders are only interested in the flow of profits (π_t); in contrast, here consumers care about the current expected value of their entire consumption prospect (given by the final product y_t, as a sum of both wages and profits). U_t can actually be interpreted as the value of an asset which gives to the owner the right to receive, as a return, the whole national income. Second, in deriving (3) we saw that the arrival of the next innovation exercises a negative effect on the market value of the incumbent (because of its ' creative destruction' effect). Conversely, from a social perspective the arrival of the successive innovation enhances unambiguously the consumers' welfare, which jumps to $U_{t+1} = \gamma U_t$, with a net collective gain equal to $U_{t+1} - U_t = (\gamma - 1)U_t$. This social gain occurs with probability λn in the unit of time, and its expected value is then

$\lambda n(U_{t+1} - U_t)$. As a result, the overall return from this 'asset' is $y_t + \lambda n(U_{t+1} - U_t)$, which must be equal to that obtained under the rate r, that is:

$$rU_t = y_t + \lambda n(U_{t+1} - U_t) \tag{15}$$

Once substituting for $y_t = A_t(L-n)^\theta$ and $U_{t+1} = \gamma U_t$, (15) can be solved for U_t and gives:

$$U_t = \frac{A_t(L-n)^\theta}{r - \lambda n(\gamma - 1)} \tag{16}$$

If λ is a given parameter, as in Aghion and Howitt (1992), then by maximizing (16) with respect to n, it is easy to obtain the socially optimal research effort:

$$n_{sp}^{(AH)} = \frac{\lambda(\gamma-1)\frac{1}{\theta}L - r}{\lambda(\gamma-1)\frac{1-\theta}{\theta}}$$

(where sp stands for 'social planner' and AH for Aghion-Howitt) to be compared with:

$$n^{(AH)} = \frac{\gamma\lambda\frac{1-\theta}{\theta}L - r}{\lambda + \gamma\lambda\frac{1-\theta}{\theta}},$$

which is the laissez-faire optimal research effort.

Our difference with respect to the standard case depicted above is concerned with the social planner's ignorance about the true value of the arrival rate λ. As in the laissez-faire problem, the selection of a specific value of λ between m and M depends on the decision rule adopted. Let us first consider the maximin criterion: a max-minimizing planner always evaluates the social welfare with respect to the worst possible state of nature. Then, by following an argument which closely resembles the one elaborated for the case of laissez-faire, the equilibrium condition (15) simply becomes:

$$rU_t = y_t + mn(U_{t+1} - U_t)$$

Once again, by substituting for $y_t = A_t(L-n)^\theta$ and $U_{t+1} = \gamma U_t$, we obtain the welfare function to be maximized:

$$U_t = \frac{A_t(L-n)^\theta}{r - mn(\gamma - 1)}$$

The maximization of this function with respect to n gives the socially optimal research effort under the MMC as:

$$n^{*}_{sp} = \frac{m(\gamma-1)\frac{1}{\theta}L - r}{m(\gamma-1)\frac{1-\theta}{\theta}},$$

which is equal to $n^{(AH)}_{sp}$ except for the presence of m in the place of λ.

Under the minimax regret criterion, with a reasoning in all respects analogous to that developed in the case of laissez-faire, the equilibrium condition can be stated as follows:

$$y_t + Mn(U_{t+1} - U_t) - rU_t = rU_t - [y_t + mn(U_{t+1} - U_t)],$$

which finally gives the following expression for U_t:

$$U_t = \frac{A_t(L-n)^{\theta}}{r - \frac{m+M}{2}n(\gamma-1)}.$$

The maximization of U_t with respect to n gives the value of the socially optimal research effort under the MMRC as:

$$n^{**}_{sp} = \frac{\frac{m+M}{2}(\gamma-1)\frac{1}{\theta}L - r}{\frac{m+M}{2}(\gamma-1)\frac{1-\theta}{\theta}}.$$

If the optimism-pessimism index is the criterion adopted, then the unknown λ is expressed through an α-weighted average of m and M, and in equilibrium it must hold:

$$y_t + [\alpha m + (1-\alpha)M]n(U_{t+1} - U_t)] = rU_t.$$

Solving the expression above for U_t gives:

$$U_t = \frac{A_t(L-n)^{\theta}}{r - [\alpha m + (1-\alpha)M]n(\gamma-1)},$$

and the optimal research effort under the OPIC is equal to:

$$n^{***}_{sp} = \frac{[\alpha m + (1-\alpha)M](\gamma-1)\frac{1}{\theta}L - r}{[\alpha m + (1-\alpha)M](\gamma-1)\frac{1-\theta}{\theta}}.$$

Finally, with the principle of insufficient reason, the arithmetic mean of m and M replaces the unknown λ, and the equilibrium condition is:

$$y_t + \frac{m+M}{2}n(U_{t+1} - U_t)] = rU_t,$$

which gives the same expression for U_t as the one we have derived in the case of the MMRC. Hence, the socially optimal research effort under the OPIC is identical to the one under the MMRC:

$n_{sp}^{****} = n_{sp}^{**}$.

By comparing the optimal laissez-faire research effort with the socially optimal one under all four decision rules (that is, n^* vs. n_{sp}^*, n^{**} vs. n_{sp}^{**} and so on) we realize that, as in Aghion-Howitt (1992), the former value can be higher or lower than the latter, and exactly for the same reason. The 'intertemporal spillover effect' and the 'appropriability effect' tend to make the laissez-faire value lower than the socially optimal value, while the 'business stealing effect' and the 'monopoly distorsion effect' operate in the opposite direction (see Aghion-Howitt (1992) for a detailed explanation of these effects): the overall final result depends on the values of the parameters involved.

3.5 Concluding Remarks

In the preceding pages we have extended a basic neo-Schumpeterian framework, as developed in Aghion and Howitt (1992), so as to encompass the investors' uncertainty about the arrival rate of innovation. As in the standard framework, the evolution of innovation in the economy follows a Poisson process. However here investors ignore the 'flow probability' of discovering an innovation, represented by the Poisson parameter λ. The unique assumption that we have imposed is concerned with their knowledge of the set of its potential values ($[m, M] \subseteq]0, \infty[$). Afterwards we have analyzed the two 'λ-sensitive decision problems' - namely, whether or not to invest in R&D, and whether to buy risk-free assets or shares of monopolistic firms - under four distinct non-probabilistic decision rules: the maximin return, the minimax regret, the optimism-pessimism index and the principle of insufficient reason. We have then found the equilibrium R&D efforts in steady-state and compared them with each other and with the Aghion-Howitt solution. We have finally carried out the comparative statics analysis and the welfare analysis, and proved that the basic neo-Schumpeterian framework, as embodied in the Aghion-Howitt's (1992) model, is robust to strong uncertainty.

Chapter 4

An Uncertainty-Based Explanation of Symmetric Growth in Neo-Schumpeterian Growth Models

4.1 Introduction

Most vertical R&D-driven growth models (such as Grossman-Helpman (1991), Segerstrom (1998), Aghion-Howitt (1998, Ch.3)) focus on the *symmetric equilibrium* in the research industries, that is,

[0] This chapter is drawn from a joint paper with Guido Cozzi and Luca Zamparelli.

on that path characterized by an equal size of R&D investments per sector. As is well known, in these models the engine of growth is technological progress, which stems from R&D investment decisions taken by profit-maximizing agents. By means of research each product line can be improved an infinite number of times and the firms manufacturing the most updated version of a product monopolize the relative market and thus earn positive profits. However, these profits have a temporary nature since any monopolistic producer is doomed to be displaced by successive improvements in her product line. The level of expected profits together with their expected duration, as compared with the cost of research, determines the profitability of undertaking R&D in each sector.

Now, the *plausibility* of the symmetric equilibrium requires that each R&D sector be equally profitable, so that the agents happen to be indifferent as to where targeting their investments. The profit-equality requirement implies two different conditions. First, the profit flows deriving from any innovation need be the same for each industry: this is guaranteed by assuming that all the monopolistic industries share the same cost and demand conditions. Second, the monopolistic position acquired by innovating needs be expected to last equally long across sectors: this requires that the agents *expect* the future amount of research to be equally distributed among the different sectors. As is well known to the reader familiar with the neo-Schumpeterian models of growth, future is allowed to affect current (investment) decisions via the forward-looking nature of the Schumpeterian 'creative destruction' effect.

Grossman and Helpman (1991, p.47) recognize the centrality of the assumption of symmetric expected R&D investments in order to justify the selection of the symmetric equilibrium: with the assumption that "the profit flows are the same for all industries [...] an entrepreneur will be indifferent as to the industry in which she devotes her R&D efforts provided that she expects her prospective leadership position to last equally long in each one. We focus hereafter on the symmetric equilibrium in which all products are targeted to the same aggregate extent. In such an equilibrium the individual entrepreneur indeed expects profit flows of equal duration in every industry and so is indifferent as to the choice of industry". Hence in this framework it is crucial to assume that an equal distribution of future R&D efforts across industries is expected.

Although expecting equal future profitability across sectors constitutes a necessary condition for each agent to choose a symmetric allocation of R&D efforts, it is however not sufficient: in fact, equal future profitability makes the investor indifferent as to where targeting research. As a result, the allocation problem of investments across product lines is indeterminate even if symmetric expectations are assumed. Notice also that the way this allocation problem is solved is not always without consequence for this class of models, as recently pointed out by Cozzi (2003). For instance in a Segerstrom's (1998) framework, because of the 'increasing complexity hypothesis', the alternative prevalence of the symmetric or asymmetric equilibrium has powerful effects on the growth rate of the economy: if indifferent agents, for a whatever reason (a 'sunspot'), are induced to allocate their investment only in a small fraction of sectors, the dynamic decreasing returns to R&D investments will imply a lower aggregate growth rate as compared with the one associated with an equal distribution of R&D efforts across all sectors. An equally relevant effect of sunspot-driven asymmetric R&D investments on steady-state growth rates reappears in the Howitt's (1999) extension to an ever expanding set of product lines (see Cozzi (2004)). Hence both solutions to the 'strong scale effect' problem (Jones (2004)) exhibit dependence of growth rates on the intersectoral distribution of R&D.

In this chapter we provide an alternative route to make the focus on the symmetric equilibrium compelling. Our basic idea is that the agent's beliefs on the future (per sector) distribution of R&D investments are characterized by uncertainty (or ambiguity), in the sense that information about that distribution is too imprecise to be represented by a (single additive) probability measure. In particular, we follow the maximin expected utility (MEU) decision rule axiomatized by Gilboa and Schmeidler (1989) (see chapter 0 for an introduction to MEU). In our framework the decision maker is then assumed to maximize her expected pay-off with respect to the R&D investment decision, while singling out the minimizing choice scenario, that is, the worst probability distribution over the future configuration of R&D investments. Unlike in Epstein and Wang (1994), here the maximin decision rule eliminates indeterminacy and makes the symmetric - and growth maximizing - allocation of R&D investment emerge as the unique equilibrium.

Importantly, our assumption on the agents' beliefs does not affect any fundamentals of the economy and is to be interpreted as a way of treating sector-specific 'extrinsic uncertainty'. Moreover, since uncertainty does not affect aggregate variables, in order to develop our argument, we don't need to introduce either the optimal consumption problem solved by households, or the profit-maximizing problem solved by firms (for which the reader is referred to Segerstrom (1998)). As the problem is that of distributing a given amount of R&D efforts across product lines, all we need is the description of the R&D sector.

Our result holds for a however small probability that a however small fraction of individual's portfolio be affected by strong uncertainty. Hence a microscopic departure from the standard treatment of extrinsic uncertainty leads to potential macroscopic growth consequences.

The rest of the chapter is organized as follows. In Section 4.2 we briefly describe the basic structure of the R&D sector, with particular reference to the Segerstrom's (1998) formalization. In Section 4.3 we explain the core of our argument, enunciate and prove the proposition. In Section 4.4 we conclude with some remarks.

4.2 The R&D Sector

In this Section we provide a description of the vertical innovation sector which is basically common to most neo-schumpeterian growth models. This sector is characterized by the efforts of R&D firms to develop better versions of the existing products in order to displace the current monopolists[1]. We assume a continuum of industries indexed by ω over the interval [0,1]. There is free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. Adopting Segerstrom's (1998) notation, any firm hiring l_j units of labor in industry ω at time t acquires the instantaneous probability of innovating $Al_j/X(\omega,t)$, where $X(\omega,t)$ is the R&D difficulty index.

Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation is $AL_I(\omega,t)/X(\omega,t) \equiv i(\omega,t)$, where $L_I(\omega,t)=\sum_j l_j(\omega,t)dj$. The parameter $X(\omega,t)$ describes the evolution of technology; as in Segerstrom (1998), we assume it to evolve in accordance with

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t)$$

where μ is a constant. However we do not impose any sign restriction on μ, in order to leave the difficulty index increasing, decreasing or remaining constant as research accumulates. In the next section we will return to this problem by specifying the range of values of μ which render our proposition significant.

Whenever a firm succeeds in innovating, she acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm: let us denote it with $v(\omega,t)$. Thus, the

[1] It seems irrelevant to our purpose to distinguish whether the monopolistic sector is that of the final goods - as in Segerstrom (1998) - or that of the intermediate ones - as in Aghion and Howitt (1998, Ch.3) and Howitt (1999).

problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize her expected profits:

$$\max_{l_j}[v(\omega,t)A/X(\omega,t)l_j - l_j]$$

which provides a finite, positive solution for l_j only when the arbitrage equation[2] $v(\omega,t)A/X(\omega,t) = 1$ is satisfied. Notice that in this case, though finite, the size of the firm is indeterminate because of the constant return research technology.

Efficient financial markets require that the stock market valuation of the firm yield an expected rate of return equal to the riskless interest rate $r(t)$. The shareholder receives a dividend of $\pi(t)dt$[3] over a time interval of length dt and the value of the monopoly appreciates by $\dot{v}(\omega,t)dt$ if no firm innovates in the unit time dt. However, if an innovation occurs, the shareholder suffers a loss of $v(\omega,t)$. It happens with probability $i(\omega,t)dt$, whereas no innovation occurs with probability $[1 - i(\omega,t)dt]$. Therefore, the expected rate of return from holding a share of monopolistic firm per unit time is

$$\frac{\pi(t) + \dot{v}(\omega,t)[1 - i(\omega,t)]}{v(\omega,t)} - i(\omega,t)$$

which needs be equal to the interest rate $r(t)$. From this equality we can derive the firm's market valuation:

$$v(\omega,t) = \frac{\pi(t)}{r(t) + i(\omega,t) - \dfrac{\dot{v}(\omega,t)}{v(\omega,t)}}$$

so that the R&D equilibrium condition is

$$\frac{\pi(t)A}{X(\omega,t)[r(t) + (1-\mu)i(\omega,t)]} = 1$$

since

[2] We consider the wage rate as the numerarie.

[3] We drop the ω argument from the profit function because, when assuming symmetric cost and demand conditions, the profit flows in each monopolistic industry coincide.

$$\frac{\dot{v}(\omega,t)}{v(\omega,t)} = \frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t).$$

The usual focus on the symmetric growth equilibrium is based on the assumption that the R&D intensity $i(\omega, t)$ is the same in all industries ω at time t and strictly positive. The suggestion of a new rationale for this symmetric behavior will be the topic of the next Section.

4.3 The re-Foundation of the Symmetric Equilibrium

We assume that the agent has a fuzzy perception of the future configuration of R&D efforts and formalize her investment strategy as an equilibrium resulting from a 'two-player zero-sum game' characterized by:

- the *minimizing* behavior of a 'malevolent Nature', which selects the prior belief associated with the 'worst possible scenario' inside a pre-specified set of priors and
- the *maximizing* behavior of the agent, whose optimal choice must take into account the worst-case strategy implemented by Nature.

Before proceeding with the analysis, let us clarify two important aspects of the model's structure. In the previous Section we have referred to the R&D firm as the one choosing the size and the distribution among sectors of R&D investments. However, R&D firms are financed by consumers' savings which are channeled to them through the stock market. Thus, since the consumer is allowed to choose the R&D sectors where to employ her savings, she ends up with being our fundamental unit of analysis. The role of the R&D firms merely becomes that of transforming these savings into research activity.

Notice also that in the basic set-up by which this work is inspired (Grossman and Helpman (1991) and Segerstrom (1998)), the agent is assumed to be risk-averse. In fact, she is assumed to be able to completely diversify her portfolio - by means of the intermediation of costless financial institutions - and then to only care about deterministic mean returns. This assumption is retained in our set-up - which allows for a whatever asymmetric configuration of investments - since, in order to carry out this diversification, it is sufficient to equally allocate investments in a non-zero measure interval of R&D sectors (and not necessarily in the whole of them). The crucial difference with respect to the standard framework is then concerned with the assumption of uncertainty-averse agents, where uncertainty only affects the mean return of the R&D investment and not its volatility, against which the agent has already completely hedged.

Assumption:

$X(\omega, 0) = X_0 \ \forall \omega \in [0, 1]$.

We assume that all industries share the same difficulty index X_0 in order to focus on the role of expectations on the kind of equilibrium that will prevail.

Our problem can be stated as follows: at time $t = 0$ an agent is asked to allocate a given amount of R&D investment among all the existing sectors. As the agent is assumed to be uncertainty-averse, in maximizing her expected pay-off she will take into account the minimizing strategy that a 'malevolent nature' will be carrying out in choosing the composition of future R&D efforts. We denote with $l_m + \alpha(\omega)$ the agent's investment in sector ω, and with $L_I + \varepsilon(\omega)$ the aggregate expected research in sector ω. l_m and L_I are, respectively, the agent's average investment per sector and the average expected research per sector. $\varepsilon(.)$ and $\alpha(.)$ represent deviations from the averages satisfying:

$$\int_0^1 \varepsilon(\omega) d\omega = 0 \qquad \int_0^1 \alpha(\omega) d\omega = 0 \qquad \text{and}$$

$$\varepsilon(\omega) > -L_I \qquad \alpha(\omega) > -l_m.$$

The presence of the two functions $\alpha(\cdot)$ and $\varepsilon(\cdot)$ is intended to allow for asymmetry both in the agent's investment and in expected research[4]. We also assume the space to be partitioned into two events: symmetric and asymmetric configuration of future R&D efforts. The first is supposed to occur with probability $1-p$ while p stands for the aggregate probability of all possible asymmetric configurations. The interval $[0,p]$ represents the unrestricted set of priors assigned to each of them. As we will see, the minimizing strategy carried out by Nature will end up with assigning probability p to the worst asymmetric configuration (which is function of the agent's choice) and 0 to all the others.

By partitioning the state space 'configuration of future investments' into the events 'asymmetric' and 'symmetric', and by assigning the probability distribution $(p, 1-p)$ to them, we have implicitly assumed that the decision maker has sufficient information to evaluate probabilistically the occurrence of both of them. In fact, what is subject to uncertainty, and then to a 'conservative assessment' through the maximin strategy, is the particular asymmetric configuration that would possibly take place among all those generated by the deviation ε. Our conclusions do not crucially hinge on this partition[5].

We can now enunciate the following:

Proposition 6 *For a however small probability (p) of deviation ($\varepsilon(\omega)$) and for a however small deviation ($\varepsilon(\omega)$) from symmetric expectation on future R&D investment, uncertainty-averse agents*

[4] These definitions imply:
$\int_0^1 [L_I + \varepsilon(\omega)]\, d\omega = L_I = L \int_0^1 [l_m + \alpha(\omega)]\, d\omega = Ll_m$
where L denotes the number of agents in the economy.
With reference to Section 2 the following relation between l_j and l_m holds:
$\int_0^1 \sum_j l_j(\omega)\, d\omega = Ll_m$.

[5] For example, it can easily be shown how our result holds for a however small perturbation of the probability distribution assigning equal probabilities to every possible configuration of future R&D investment across sectors.

who adopt a maximin strategy to solve their investment allocation problem, choose a symmetric investment strategy, i.e. $l_m + \alpha(\omega) = l_m$ $\forall\omega \in [0,1]$. *Their optimal investment choice makes them expect a symmetric distribution of future R&D effort among sectors:* $L_I + \varepsilon(\omega) = L_I$ $\forall\omega \in [0,1]$.

Proof. $\max\limits_{\alpha(.)} \left[\min\limits_{\varepsilon(.)} \int_0^1 [l_m + \alpha(\omega)] \frac{A}{X_0} v(\omega) d\omega \right]$

$$\text{sub} \quad \int_0^1 \varepsilon(\omega)d\omega = 0 \qquad ; \qquad \int_0^1 \alpha(\omega)d\omega = 0$$

where now

$$v(\omega) = p\frac{\pi}{r+\frac{A}{X_0}[L_I+\varepsilon(\omega)](1-\mu)} + (1-p)\frac{\pi}{r+\frac{A}{X_0}L_I(1-\mu)}$$

Then the problem is equivalent to:

$$\max_{\alpha(.)} \left[\min_{\varepsilon(.)} \int_0^1 [l_m + \alpha(\omega)] \left(p\frac{\frac{A}{X_0}\pi}{r+\frac{A}{X_0}[L_I+\varepsilon(\omega)](1-\mu)} + (1-p)\frac{\frac{A}{X_0}\pi}{r+\frac{A}{X_0}L_I(1-\mu)} \right) d\omega \right] =$$

$$= (1-p)\frac{l_m\frac{A}{X_0}\pi}{r+\frac{A}{X_0}L_I(1-\mu)} + p\max_{\alpha(.)} \left[\min_{\varepsilon(.)} \int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r+\frac{A}{X_0}[L_I+\varepsilon(\omega)](1-\mu)} d\omega \right].$$

In order to solve the maxmin problem above, we only need to consider the second term, since the first term is constant, that is:

$$\max_{\alpha(.)} \left[\min_{\varepsilon(.)} \int_0^1 [l_m + \alpha(\omega)] \frac{\dfrac{A}{X_0}\pi}{r + \dfrac{A}{X_0}\left[L_I + \varepsilon(\omega)\right](1-\mu)} d\omega \right]$$

$$\text{sub} \quad \int_0^1 \varepsilon(\omega)d\omega = 0 \qquad ; \qquad \int_0^1 \alpha(\omega)d\omega = 0.$$

Notice that this is valid for however small probability p. We restrict our attention to the case: $0 < v(\omega) < +\infty$. As at time $t = 0$, A, X_0, π, r are assumed to be positive constants, that condition is equivalent to imposing $r + \dfrac{A}{X_0}\left[L_I + \varepsilon(\omega)\right](1-\mu) > 0 \Longleftrightarrow \mu < 1 + \dfrac{rX_0}{A\left[L_I + \varepsilon(\omega)\right]}$. Moreover we want $\mu \neq 1$, otherwise the creative destruction effect disappears from $v(\omega)$ making our problem independent of expectations.

Let us suppose the following (*not necessarily minimizing*) expected reaction function of Nature: $\varepsilon(\omega) = t\alpha(\omega)$ with $t > 0$. Notice that, as t tends to zero, the deviation in each sector can be made arbitrarily small independently of the population size. The objective function then becomes:

$$\int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + t\alpha(\omega)](1-\mu)} d\omega$$

Since $[l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + t\alpha(\omega)](1-\mu)}$ is a strictly concave function of $\alpha(\omega)$[6], by Jensen's inequality its integral

$$\int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + t\alpha(\omega)](1-\mu)} d\omega$$

is maximized by $\alpha(\omega) = 0$ for all $\omega \in [0,1]$. Therefore, if $\alpha(\omega) \neq 0$ in a non-zero measure subset of $[0,1]$, it follows:

$$\min_{\varepsilon(.)} \int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + \varepsilon(\omega)](1-\mu)} d\omega \leq \int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + t\alpha(\omega)](1-\mu)} d\omega$$

$$< \int_0^1 [l_m + \alpha(\omega)] \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}L_I(1-\mu)} d\omega = \frac{\frac{A}{X_0}\pi l_m}{r + \frac{A}{X_0}L_I(1-\mu)} \qquad \text{(D)}$$

which is instead attained if $\alpha(\omega) = 0$ for all $\omega \in [0,1]$ (almost everywhere). If indeed $\alpha(\omega) = 0$ for all $\omega \in [0,1]$ (almost everywhere), Jensen's inequality implies:

$$\min_{\varepsilon(.)} \int_0^1 l_m \frac{\frac{A}{X_0}\pi}{r + \frac{A}{X_0}[L_I + \varepsilon(\omega)](1-\mu)} d\omega = \frac{\frac{A}{X_0}\pi l_m}{r + \frac{A}{X_0}L_I(1-\mu)} \qquad \text{(E)}$$

which is attained if $\varepsilon(\omega) = 0$ for all $\omega \in [0,1]$ (almost everywhere).

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[6] It can be easily verified that the concavity of the function is ensured by the condition $\mu < 1 + \frac{rX_0}{A[L_I + \varepsilon(\omega)]}$, that we have already imposed.

Therefore the worst harm Nature can inflict to the agent in the case $\alpha(\omega) = 0$ is always better for the agent than the worst harm Nature can inflict in the case $\alpha(\omega) \neq 0$. This fact is represented by the expression (E) and the disequality (D): if the agent chooses to play a symmetric strategy ($\alpha(\omega) = 0$), the minimizing strategy of Nature is also the symmetric one ($\varepsilon(\omega) = 0$), as stated in (E). On the contrary, if the agent chooses to play a *whatever* asymmetric strategy ($\alpha(\omega) \neq 0$ in a non-zero measure subset of $[0, 1]$), then the minimizing strategy of Nature, *whatever it is*, brings to the agent a pay-off which is strictly lower than the one she gets by playing the symmetric strategy, as stated in (D). The crucial point is that we do not need to compute the exact minimizing strategy of Nature for every agent's choice in order to draw the conclusion that the symmetric portfolio - and zero measure deviations from it - is the max-minimizing strategy of the agent. Notice also that we have used the particular reaction function $\varepsilon(\omega) = t\alpha(\omega)$ in order to state the disequality (D), and that this function does not result in any loss of generality of the proof. Finally, since this result holds for any $t > 0$ and $0 < p < 1$, the statement follows for however small deviations and their probabilities. ■

Then, even if the agent is 'almost sure' ($p \to 0$) of facing a symmetric configuration of future investments (which would leave her in a position of indifference in her current allocation problem), the mere possibility of a slightly different configuration ($\varepsilon \to 0$) makes her strictly prefer to equally allocate her investments across sectors. This occurs because, whenever the agent evaluates an asymmetric allocation of her current investments, she will always be induced to expect the worst configuration of future investments inside the ε-generated set.

4.4 Concluding Remarks

In the neo-schumpeterian growth models the existence of the creative destruction effect implies that expectations on future R&D investments affect the allocation of current ones. Therefore the usual focus on the symmetric equilibrium in the vertical research sector relies on the assumption of a symmetric expected per-sector distribution of R&D expenditure. However, in making the agents indifferent as to where targeting their investments, this assumption is not sufficient to univocally pin down the symmetric structure of R&D efforts: actually symmetric expectation on future R&D leaves the current composition of R&D investments indeterminate, with potentially large effects on growth rates.

We have shown that a possible way out of this indeterminacy is that of assuming uncertainty on the future configuration of R&D investments and max-minimizing agents in the face of this uncertainty. Under this assumption, indeterminacy vanishes and the symmetric allocation of the vertical research expenditures comes out as the unique optimal solution.

Chapter 5

Uncertainty Averse Agents in a Quality-Ladder Growth Model with Asymmetric Fundamentals

5.1 Introduction

Quality ladder growth models, such as Grossman-Helpman (1991) and Segerstrom (1998), focus on the role of technical progress as the main source of economic growth. In this class of models technical change is the outcome of R&D investment decisions taken by profit maximizing firms. Any product line can be improved an infinite number of times by means of research, and the firms manufacturing the most updated version of a product monopolize the relative market and earn positive profits. However, these profits have a temporary nature, because they last until the next improvement in the same product line occurs. In the neo-Schumpeterian literature this effect is commonly referred to as 'creative destruction'. The profitability of undertaking R&D in each sector depends on three sets of conditions: the magnitude of the profit flows associated with any monopolistic position, the difficulty of acquiring such a position (i.e., the probability of innovating) and its expected duration. We can refer to the first two sets of conditions as the fundamentals of the economy, as they respectively depend on the costs and demand conditions of the commodity sector and on the technology of the research industry. On the other hand, the expected duration of the profit flows in a particular R&D sector depends on the agents' expectations on the future amount of research which will be carried out in that sector. That is, future affects current investment decisions to the extent that agents anticipate the 'creative destruction' effect.

In the standard literature the economy is assumed to have symmetric fundamentals: any sector shares the same cost and demand conditions, as well as the same probabilty of innovating. In order to equalize the overall profitability across the R&D sectors, symmetric expectations on future R&D investments are also assumed. With these assumptions, the focus on the symmetric equilibrium in R&D investments is made plausible.

In this Chapter we generalize a standard quality-ladder growth model, with particular reference to Segerstrom (1998), by assuming asymmetric fundamentals[1]. We find that the creative destruction effect due to the expectations on future research is the sole responsible for equalizing the expected returns in R&D; such equalization is required if we want to derive positive R&D efforts in each industry. Moreover, as the rational expectations equilibrium requires the equality between expectations on research investments and their actual values, the actual investments in R&D will equalize the returns across sectors. Still, it does not seem that this RE equilibrium is uniquely pinned down,

[1] See also Zamparelli (2003).

since equal future profitability makes the investor *indifferent* as to where targeting research. As a result, when the expectations on future research equalize the expected returns for all R&D soctors, the allocation problem of investments across product lines is *indeterminate*. Our main purpose is then to micro-found the equilibrium which equalizes returns across sectors[2]. We assume that, while agents have perfect knowledge of the fundamentals of the economy, their beliefs on the future (per sector) distribution of R&D investments are characterized by strong uncertainty. To formalize the idea that information about that distribution is too vague to be represented by a - single additive - probability measure, here we will follow once again the maximin expected utility (MEU) model axiomatized by Gilboa and Schmeidler (1989) (see chapter 0 of this book for a basic introduction). This means, in our framework, that the decision maker will be assumed to maximize her expected pay-off with respect to the R&D investment decision, while singling out the worst probability distribution (inside her set of multiple priors) over the future configuration of R&D investments. With this assumption we provide a re-foundation of the rational expectations equilibrium, in which actual and expected R&D efforts are equal among each others and are such that returns are equalized across sectors.

The rest of the chapter is organized as follows. In Section 5.2 we introduce the asymmetric model. In Section 5.3 we explain the core of our argument, enunciate and prove the propositions. In Section 5.4 we conclude with some remarks.

[2] Notice that this is a problem analogous to the one that we encountered in the previous chapter with reference to the standard symmetric models. As we will see, the solution to this problem is also very similar.

5.2 The Model

As usual, we assume a continuum of industries producing final goods indexed by $\omega \in [0,1]$. Asymmetry is introduced by assuming industry-specific:

- quality-jumps ($\lambda(\omega)$) in technical progress
- arrival rates of innovation ($A(\omega)$)
- weights of the Cobb-Douglas utility function ($\alpha(\omega)$)
- unit production costs in the manufacturing sector (l_ω)

We also suppose a fixed number of dynastic households (normalized to one), whose members grow at the constant rate $n > 0$. Each member shares the same intertemporally additively separable utility $u(t)$ and is endowed with a unit of labor she supplies inelastically. Therefore each household chooses its optimal consumption path by maximizing the discounted utility:

$$U \equiv \int_0^\infty L(0)e^{-(\rho-n)t} \log u(t)dt \tag{1}$$

where $L(0) \equiv 1$ is the initial population and $\rho > n$ is the common rate of time preferences. With $\log u(t) \equiv \int_0^1 \alpha(\omega) \log \left[\sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j,\omega,t) \right] d\omega$,

the static maximization problem can be represented as:

$$\max \int_0^1 \alpha(\omega) \log \left[\sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j,\omega,t) \right] d\omega \tag{2}$$

$$\text{s.t. } E(t) = \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,t)} p(j,\omega,t) d(j,\omega,t) \right], \tag{3}$$

where $d(j,\omega,t)$ and $p(j,\omega,t)$ denote, respectively, consumption and price of good ω of quality j at time t. $E(t)$ is the total expenditure at time t. $j^{\max}(\omega,t)$ denotes the highest quality reached by product ω at time t. We impose $\int_0^1 \alpha(\omega)d\omega = 1$ to represent the homogeneity of degree one of the utility function.

At each point in time consumers maximize static utility by spreading their expenditure across sectors proportionally to the utility contribution of each product line ($\alpha(\omega)$), and by only purchasing in each line the product with the lowest price per unit of quality. As usual in quality-ladder models, because of the Bertrand competition in the manufacturing sector, this product is the one indexed by $j^{\max}(\omega, t)$. Then, the individual static demand functions are:

$$d(j, \omega, t) = \begin{cases} \dfrac{\alpha(\omega)E(t)}{p(j, \omega, t)} & \text{for } j = j^{\max}(\omega, t) \\ \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Moreover, since only the $j^{\max}(\omega, t)$ quality product is actually purchased, in what follows it will be: $\sum\limits_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) = \lambda^{j^{\max}(\omega,t)}(\omega)$.

Substituting (4) into (2) and (2) into (1), we obtain the intertemporal maximum problem

$$\max_E U = \int_0^\infty e^{-(\rho-n)t} \left[\log E(t) + \int_0^1 \alpha(\omega) \left[\log \alpha(\omega) + \log \lambda^{j^{\max}(\omega,t)}(\omega) - \log p(j, \omega, t)\right] d\omega\right] dt$$

$$\text{s.t. } \int_0^\infty e^{-\int_0^t [r(s)-n]ds} E(t)dt \leq A(0),$$

where $r(s)$ is the instantaneous interest rate at time s and $A(0)$ is the present value of the stream of incomes plus the value of initial wealth at time $t = 0$. The solution to this problem obeys the differential equation:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \tag{5}$$

Each good is produced by only employing labor through a constant return to scale technology. In order to produce one unit of good ω, firms hire l_ω units of labor regardless of quality. The Bertrand competition implies that the quality leader monopolizes her relative market, and that the limit price she can charge is $p\left[j^{\max}(\omega, t), \omega, t\right] = \lambda(\omega)wl_\omega$. Then the profit flows in each sector are:

$$\pi(\omega, t) = \frac{\lambda(\omega) - 1}{\lambda(\omega)}\alpha(\omega)E(t)L(t).$$

Firms can engage in R&D to develop better versions of the existing products in order to displace the current monopolists. As usual, we assume free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. Any firm hiring l_k units of labor in industry ω at time t acquires the instantaneous probability of innovating $A(\omega)l_k/X(\omega,t)$, where $X(\omega,t)$ is the R&D difficulty index, which is introduced to rule out the 'scale effect'. Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation (or research intensity) is:

$$\frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} \equiv i(\omega,t), \tag{6}$$

where $L_I(\omega,t) = \sum_k l_k(\omega,t)$. As R&D proceeds, its difficulty index $X(\omega,t)$ is assumed to increase over time (so as to rule out the scale effect). With reference to Segerstrom (1998), we model the increasing complexity hypothesis according to what is usually called 'TEG specification'[3]:

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t), \tag{TEG}$$

where μ is a positive constant.

Whenever a firm succeeds in innovating, she acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm: $v(\omega,t)$. Thus, the problem faced by a R&D firm is that of choosing the amount of labor input in order to maximize her expected profits:

$$\max_{l_k}\left[\frac{v(\omega,t)A(\omega)}{X(\omega,t)}l_k - l_k\right].$$

The expression above provides a finite, positive solution for l_k only when the arbitrage equation[4]:

[3]The acronym TEG stands for 'Temporary effects on growth' of policy measures such as subsidies and taxes. Useful surveys on the scale effect problem and on the way it has been solved are Dinopoulos and Thompson (1999) and Jones (1999 and 2003).

[4]We consider the wage rate as the numerarie.

$$\frac{v(\omega,t)A(\omega)}{X(\omega,t)} = 1$$

is satisfied. Efficient financial markets require that the stock market valuation of the firm yields an expected rate of return equal to the riskless interest rate $r(t)$. Then, the firm's market valuation is:

$$v(\omega,t) = \frac{\pi(\omega,t)}{r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}},$$

so that the R&D equilibrium condition is:

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}\right]} = 1. \qquad (7)$$

Since in each industry the market demand $D(\omega,t) = \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)l_\omega}$ requires $D(\omega,t)l_\omega$ units of labor in order to be produced, the total employment in the manufacturing sector is given by $\int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)}d\omega$. Then, the labor market-clearing condition implies:

$$L(t) = \int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)}d\omega + \int_0^1 L_I(\omega,t)d\omega \qquad (8)$$

where $\int_0^1 L_I(\omega,t)d\omega$ is the total employment in the research sector.

The rational expectations equilibrium requires that, at each point in time, the expected $L_I(\omega,t)$, on which the 'creative destruction effect' depends, be equal to its actual value. This would allow us to consider $L_I(\omega,t)$ in (7) and (8) as the same variable and to derive the equilibrium values of $E(t)$ and $L_I(\omega,t)$, which contemporarily verify the system made up of (7) and (8). We could then solve for the steady-state values of E and $L_I(\omega,t)$ and obtain[5]:

$$E = \frac{\frac{\mu}{n}\rho + 1 - \mu}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}$$

and

[5] For the derivation of these results see Zamparelli (2003).

$$L_I(\omega,t) = L(t)\frac{\alpha(\omega) - \dfrac{\alpha(\omega)}{\lambda(\omega)}}{\left(\dfrac{\mu}{n}\rho - \mu\right)\int\limits_0^1 \dfrac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}$$

In the next Section our assumption on the agents' beliefs will allow for separating the actual and the expected R&D efforts.

5.3 The Equilibrium with Uncertainty Averse Agents

In deriving our result we will assume that the agent's beliefs on the future composition of R&D efforts are 'strongly' uncertain. This assumption essentially incorporates the idea that the agent ignores both the (true) future distribution of R&D investments across sectors and a precise (single additive) probability measure over all possible (investment) distributions. In our allocation problem, a preference for certainty (or 'uncertainty aversion') is introduced and basically formalized as in the MEU model (Gilboa and Schmeidler (1989)). As we clarified in chapter 0, in representing subjective beliefs, MEU suggests to replace the standard single prior with a closed and convex set of priors. The choice among alternative acts is determined by a maximin strategy. For each act the agent first computes the expected utilities with respect to each single prior in the set and picks up the minimal value. Finally she compares all these values and singles out the act associated with the highest (minimal) expected utility.

Before proceeding with the analysis, let us first make an important remark. In the previous Section we have referred to the R&D firm as the one choosing the size and the distribution among sectors of R&D investments. However, R&D firms are financed by consumers' savings, which are channeled to them through the stock market. Thus, since the consumer is allowed to choose the R&D sectors where to employ her savings, she ends up with being our fundamental unit of analysis.

The role of the R&D firms merely becomes that of transforming these savings into research activity.

The agent is asked to allocate a certain amount of R&D investment among all the existing sectors. In maximizing her expected pay-off she will take into account the minimizing strategy that a 'malevolent nature' will be carrying out in choosing the composition of future R&D efforts. The formalization of the investment allocation problem closely resembles the one we have seen in the preceding Chapter. We denote with $l_m + \gamma(\omega)$ the agent's investment in sector ω, and with $L_I + \varepsilon(\omega)$ the aggregate expected research in sector ω. l_m and L_I are, respectively, the agent's average investment per sector and the average expected research per sector. $\varepsilon(.)$ and $\gamma(.)$ represent deviations from the averages satisfying:

$$\int_0^1 \varepsilon(\omega)d\omega = 0 \qquad \int_0^1 \gamma(\omega)d\omega = 0 \qquad \text{and}$$

$$\varepsilon(\omega) > -L_I \qquad \gamma(\omega) > -l_m.$$

The presence of the two functions $\gamma(\cdot)$ and $\varepsilon(\cdot)$ is intended to allow for asymmetry both in the agent's investment and in expected research[6]. We also assume the space to be partitioned into two events: symmetric and asymmetric configuration of future R&D efforts. The first is supposed to occur with probability $1-p$, while p stands for the aggregate probability of all possible asymmetric configurations. The interval $[0,p]$ represents the unrestricted set of priors assigned to each of them. As we will see, the minimizing strategy carried out by Nature will end up with assigning probability p to the worst asymmetric configuration (which is function of the agent's choice) and 0 to all the others. We can now enunciate the following:

[6] These definitions imply:

$$\int_0^1 [L_I + \varepsilon(\omega)]\, d\omega = L_I = L\int_0^1 [l_m + \alpha(\omega)]\, d\omega = Ll_m$$

where L denotes the number of agents in the economy.

With reference to Section 2 the following relation between l_j and l_m holds:

$$\int_0^1 \sum_j l_j(\omega)\, d\omega = Ll_m.$$

Proposition 7 *For a however small probability (p) of deviation from the 'return-equalizing' expectations on future R&D investment, uncertainty averse agents choose the following investment strategy:*

$$l_m + \gamma(\omega) = l_m \frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} \quad \forall\omega \in [0,1].$$

Their expectation on the distribution of future R&D effort among sectors is:

$$L_I + \varepsilon(\omega) = \frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} \left(L_I^e(t) + \frac{r(t)}{(1-\mu)} \int_0^1 \frac{X(\omega,t)}{A(\omega)} d\omega \right) - \frac{r(t)}{(1-\mu)} \frac{X(\omega,t)}{A(\omega)} \quad \forall\omega \in [0,1].$$

Proof. See Appendix A1. ■

This result proves to be relevant as soon as we turn to the *steady-state* equilibrium. In steady-state all endogenous variables grow at constant rates, $\frac{\dot{X}(\omega,t)}{X(\omega,t)} \equiv \mu i(\omega) = n$ and, as $\frac{\dot{E}(t)}{E(t)} = 0$, $r(t) = \rho$. It is easy to show, by substituting for $L_I^e(t) + \varepsilon(\omega,t)$ into $v(\omega,t)$, that the R&D returns $(v(\omega,t)A(\omega)/X(\omega))$ are equalized across sectors. In particular[7]:

$$v(\omega,t)A(\omega)/X(\omega) = \left(\frac{L_I^e(t)}{EL(t)} \left(1 - \mu + \frac{\rho\mu}{n} \right) \right)^{-1}.$$

Now, by using the arbitrage equation of any of the R&D sectors (equation (7)), we can solve for $L_I^e(t)$:

$$L_I^e(t) = \frac{EL(t)\left(1 - \frac{\alpha(\omega)}{\lambda(\omega)}\right)}{\frac{\mu}{n}\rho + 1 - \mu} \tag{9}$$

The market-clearing condition is:

$$L(t) = \int_0^1 \frac{\alpha(\omega)EL(t)}{\lambda(\omega)} d\omega + L(t) \int_0^1 \left[l_m + \gamma(\omega,t) \right] d\omega$$

Dividing by $L(t)$, it can be written as:

$$1 = E \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + l_m \tag{10}$$

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[7] The derivation of the following result is worked out in Appendix A2.

The steady-state resource (10) and arbitrage (9) equations allow us to find the equilibrium values of l_m and E.

$$E = \frac{\frac{\mu}{n}\rho + 1 - \mu}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}$$

$$l_m = \frac{1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}.$$

We can now state the following:

Proposition 8 *If agents are uncertain on the future R&D per-sector distribution and adopt a maximin strategy to solve their R&D allocation problem, in steady state the actual investments make the R&D returns equal across sectors. The values of these investments are:*

$$l_m + \gamma(\omega, t) = \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}.$$

Proof. See Appendix A3. ∎

5.4 Concluding Remarks

In this Chapter we have first extended a basic quality-ladder growth model (see Grossman-Helpman (1991) and Segerstrom (1998)), to embrace the possibility of asymmetric fundamentals across the existing sectors. In this model, developed in Section 5.2, we have then pointed out that, in order for each sector to be characterized by positive research effort in steady-state, it is necessary that expectations on future research make returns equal across sectors. However, we have argued, if these returns are perfectly equalized, the investor happens to be totally indifferent as to where targeting research. Hence, the investment allocation problem across sectors is indeterminate. As we have seen in the previous Chapter, this problem of indeterminacy also exists in the standard (symmetric) framework and, in that case, we have provided a solution based on the agents' uncertain beliefs about the exact distribution of R&D investments across the exisiting sectors. Even if it is implemented in a more complicated framework and implies different technicalities (see the Appendix), the solution we have proposed here is much in the same spirit as that of the preceding Chapter. We have assumed the agents to be uncertain about the future configuration of R&D investments in the sense of Gilboa and Schmeidler (1989), and to 'guard themselves' against uncertainty by using a maximin strategy. As proven in Section 5.3, it is sufficient a however small probability of expecting a non-return-equalizing future configuration of investments to induce the agents to make the 'right choice', and hence, to univocally pin down the (rational expectations) return-equalizing equilibrium.

5.5 Appendix A1

Proof of proposition 11. $\max_{\gamma(\cdot)} \left[\min_{\varepsilon(\cdot)} \int_0^1 [l_m + \gamma(\omega,t)]\, v(\omega,t) \frac{A(\omega)}{X(\omega,t)} d\omega \right]$

s.t. $\int_0^1 \gamma(\omega,t) d\omega = \int_0^1 \varepsilon(\omega,t) d\omega = 0$

s.t. $\varepsilon(\omega,t) > -L_I^e; \quad \gamma(\omega,t) > -l_m.$

where $v(\omega,t) \equiv \dfrac{\pi(\omega,t)}{r(t) - \dfrac{\dot{v}(\omega,t)}{v(\omega,t)} + \dfrac{A(\omega)}{X(\omega,t)} [L_I^e(t) + \varepsilon(\omega,t)]}$

Under TEG specification $\dfrac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu \dfrac{A(\omega)}{X(\omega,t)} [L_I^e(t) + \varepsilon(\omega,t)]$. Moreover, as by differentiating (7) with respect to time, we obtain: $\dfrac{\dot{v}(\omega,t)}{v(\omega,t)} = \dfrac{\dot{X}(\omega,t)}{X(\omega,t)}$, then

$$v(\omega,t) \equiv \frac{\pi(\omega,t)}{r(t) + (1-\mu) \dfrac{A(\omega)}{X(\omega,t)} [L_I^e(t) + \varepsilon(\omega,t)]}$$

Notice that the return of any investment is industry-specific $\left(v(\omega,t) \dfrac{A(\omega)}{X(\omega,t)} \right)$ with probability p, while it is constant across industry with probability $(1-p)$ (let us define this constant value as $q(t)$). Then the problem is equivalent to:

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1[l_m+\gamma(\omega,t)]\left(p\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}+(1-p)q(t)\right)d\omega\right]$$

$$=(1-p)q(t)+p\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1[l_m+\gamma(\omega,t)]\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}d\omega\right]$$

which admits the same solution as:

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1[l_m+\gamma(\omega,t)]\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\dfrac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}d\omega\right].$$

Notice that this is valid for a however small probability p. Given these conditions, we first solve for the minimization problem:

$$\min_{\varepsilon(\cdot)}\int_0^1\frac{[l_m+\gamma(\omega,t)]\,\pi(\omega,t)}{\dfrac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))}d\omega$$

s.t. $\int_0^1\varepsilon(\omega,t)d\omega=0$

We set $e(\omega,t)=\int_0^\omega\varepsilon(s,t)ds$; then $e'(\omega,t)=\varepsilon(\omega,t)\ \forall\omega\in[0,1]$ and the minimization problem $(\mathrm{P_{min}})$ can be expressed as:

$$\min_{e'(\cdot)}\int_0^1 G(e')d\omega$$

s.t. $e(0)=0$; $e(1)=0$

where $G(e')=\dfrac{[l_m+\gamma(\omega,t)]\,\pi(\omega,t)}{\dfrac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))}$

This is the simplest problem of calculus of variations. Since under the conditions specified above $G(e')\in C^2$, we can apply the Euler theorem stating that:

if $G(e,e',\omega)\in C^2$ and e^* is optimal and C^1, then e^* must necessarily solve:

$$G_e-\frac{d}{d\omega}G_{e'}=0 \qquad \text{(E-E)}$$

As in our case G does not depend on e, $G_e=0$; hence E-E becomes:

$$\frac{d}{d\omega}G_{e'} = 0,$$

implying that:

$$G_{e'} \equiv G_{\varepsilon} = -\frac{\pi(\omega,t)[l_m+\gamma(\omega,t)]}{\left[\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))\right]^2}$$

be constant with respect to ω.Hence:

$$\frac{\pi(\omega,t)\left[l_m+\gamma(\omega,t)\right]}{\left[\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))\right]^2} = k_1$$

where k_1 is a real constant. Now we solve the expression above for $\varepsilon(\omega)$:

$$\varepsilon(\omega) = \sqrt{\frac{\pi(\omega,t)\left[l_m+\gamma(\omega,t)\right]}{k_1(1-\mu)}} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t) \qquad \text{(R-F)}$$

This function can easily be interpreted as the *reaction function* (R-F) of the 'nature' to the agent's decision. We can now plug it into the maximization problem ($P_{\max}$) and solve for γ:

$$\max_{\gamma(.)}\int_0^1 [l_m+\gamma(\omega,t)]\frac{\pi(\omega,t)}{\sqrt{\frac{\pi(\omega,t)\left[l_m+\gamma(\omega,t)\right](1-\mu)}{k_1}}}d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t)d\omega = 0$$

Rearranging, this problem becomes:

$$\max_{\gamma(.)}\int_0^1 [l_m+\gamma(\omega,t)]^{\frac{1}{2}}\left(\pi(\omega,t)k_1/(1-\mu)\right)^{\frac{1}{2}}d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t)d\omega = 0.$$

Again, we solve $P_{\max}$ as a problem of calculus of variations. By setting $c(\omega,t) = \int_0^{\omega}\gamma(s,t)ds$, so that $c'(\omega,t) = \gamma(\omega,t)$, $P_{\max}$ becomes:

$$\max_{c'} \int_0^1 F(c')d\omega$$

sub $c(0) = 0$; $c(1) = 0$

where $F(c') \equiv F(\gamma) = [l_m + \gamma(\omega,t)]^{\frac{1}{2}} [\pi(\omega,t)k_1]^{\frac{1}{2}}$

With the same reasoning as before, the Euler Equation $F_c - \dfrac{d}{d\omega}F_{c'} = 0$ implies:

$$F_{c'} \equiv F_\gamma = -\frac{(\pi(\omega,t)k_1)^{\frac{1}{2}}}{2[l_m + \gamma(\omega,t)]^{\frac{1}{2}}} = -k_2 \qquad [\mathrm{F}_\gamma]$$

where $k_2 \in R_+$. From F_γ we can derive the expression for $\gamma(\omega,t)$:

$$\gamma(\omega,t) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m \qquad \text{(A1)}$$

Plugging it into the (R-F), we get:

$$\varepsilon(\omega) = \sqrt{\frac{\pi(\omega,t)\left[l_m + \dfrac{\pi(\omega,t)k_1}{4k_2^2} - l_m\right]}{k_1}} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t) =$$

$$= \frac{\pi(\omega,t)}{2k_2} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t) \qquad \text{(A2)}$$

Now we can use the two conditions imposed by the constraints in order to find the constants k_1, k_2:

$$\int_0^1 \gamma(\omega,t)d\omega = 0 \quad \Longleftrightarrow \quad \int_0^1 \left[\frac{\pi(\omega,t)k_1}{4k_2^2} - l_m\right] d\omega = 0$$

Hence:

$$k_1 = \frac{4k_2^2(1-\mu)l_m}{\int_0^1 \pi(\omega,t)d\omega} \qquad \text{(A3)}$$

and

$$\int_0^1 \varepsilon(\omega,t)d\omega = 0 \quad \Longleftrightarrow \quad \int_0^1 \left[\frac{\pi(\omega,t)}{2(1-\mu)k_2} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t)\right] d\omega = 0$$

whence:

$$k_2 = \frac{\int_0^1 \pi(\omega,t)d\omega}{2(1-\mu)\left[\dfrac{r(t)}{(1-\mu)}\int_0^1 \dfrac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]} \qquad \text{(A4)}$$

Substituting (A4) into (A3), we obtain:

$$k_1 = \frac{l_m \int_0^1 \pi(\omega,t)d\omega}{(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]^2} \qquad \text{(A5)}$$

Finally we can plug (A4) and (A5) into (A1) and (A2) in order to get the optimal pair $\gamma^*(\omega,t)$, $\varepsilon^*(\omega,t)$:

$$\gamma^*(\omega) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m = \frac{\pi(\omega,t)\frac{l_m \int_0^1 \pi(\omega,t)d\omega}{(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]^2}}{\frac{\int_0^1 \pi(\omega,t)d\omega}{2(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]}} - l_m$$

$$= l_m\left[\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} - 1\right]$$

$$\varepsilon^*(\omega) = \frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right] - \frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)} - L_I^e(t) =$$

$$= L_I^e(t)\left[\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} - 1\right] + \frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega - \frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)}$$

from which we can easily obtain the result of proposition 11. ■

5.6 Appendix A2

We derive the steady state value of $v(\omega,t)A(\omega)/X(\omega)$ by substituting for $L_I^e(t) + \varepsilon(\omega,t)$ given in proposition 11.

$$\frac{A(\omega)v(\omega)}{X(\omega)} = \frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\rho + \frac{A(\omega)L_I^e(t) + \varepsilon(\omega,t)}{X(\omega,t)}(1-\mu)} =$$

$$= \frac{\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}\rho+(1-\mu)\left(\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}\left(L_I^e(t)+\frac{\rho}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega\right)-\frac{\rho}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)}\right)} =$$

$$= \frac{\pi(\omega,t)}{\frac{\mu}{n}\rho\left(L_I^e(t)+\varepsilon(\omega,t)\right)+(1-\mu)\left(\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}\left(L_I^e(t)+\frac{\rho}{(1-\mu)}\frac{\mu}{n}L_I^e(t)\right)-\frac{\rho}{(1-\mu)}\frac{\mu}{n}\left(L_I^e(t)+\varepsilon(\omega,t)\right)\right)} =$$

(where we substituted for the steady state value of $X(\omega,t) = \frac{\mu}{n}A\left(L_I^e(t)+\varepsilon(\omega,t)\right))$

$$= \frac{\pi(\omega,t)}{(1-\mu)\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}L_I^e(t)+\frac{\rho\mu}{n}\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}L_I^e(t)} =$$

$$= \left(\frac{L_I^e(t)}{EL(t)}\left(1-\mu+\frac{\rho\mu}{n}\right)\right)^{-1}.$$

5.7 Appendix A3

Proof of Proposition 12. By plugging the mean value l_m into the expression for $l_m+\gamma(\omega)$ that we have found in Proposition 1, we obtain:

$$l_m+\gamma(\omega) = l_m\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} = \frac{1-\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\left(\frac{\mu}{n}\rho-\mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega+1}.\frac{\alpha(\omega)-\frac{\alpha(\omega)}{\lambda(\omega)}}{1-\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega} = \frac{\alpha(\omega)-\frac{\alpha(\omega)}{\lambda(\omega)}}{\left(\frac{\mu}{n}\rho-\mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega+1}$$

These are the steady state per sector investments. As these values are exactly the same as those we derived for the asymmetric case under certainty conditions, we have re-founded the asymmetric equilibrium which equalizes the R&D investments across sectors. ■

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